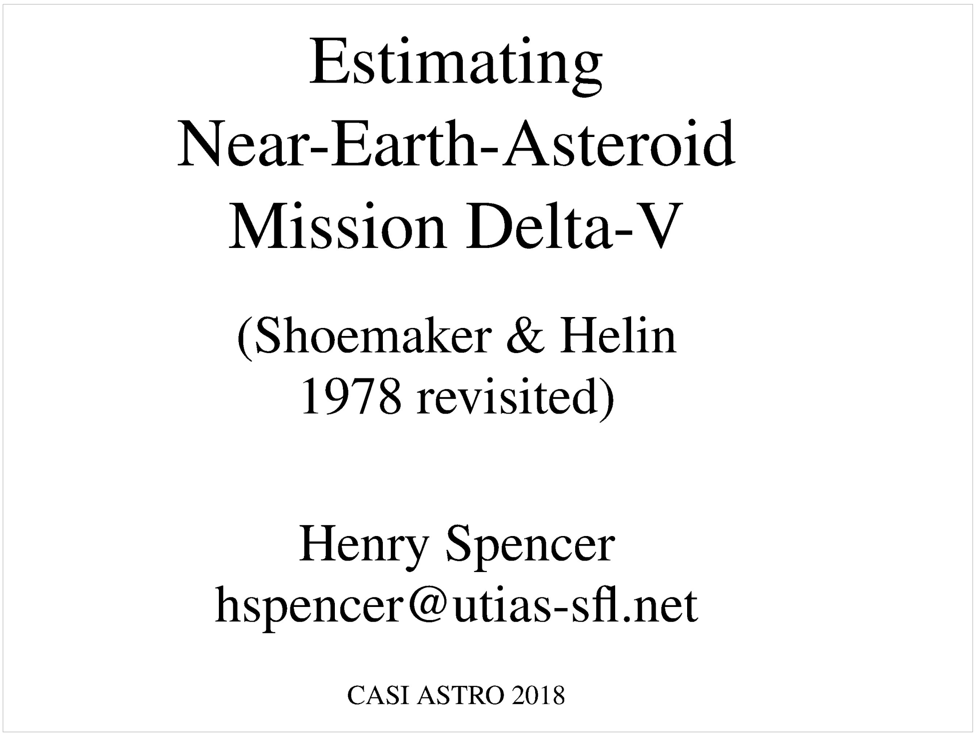

Estimating Near-Earth-Asteroid Mission Delta-V

(Shoemaker & Helin
1978 revisited)

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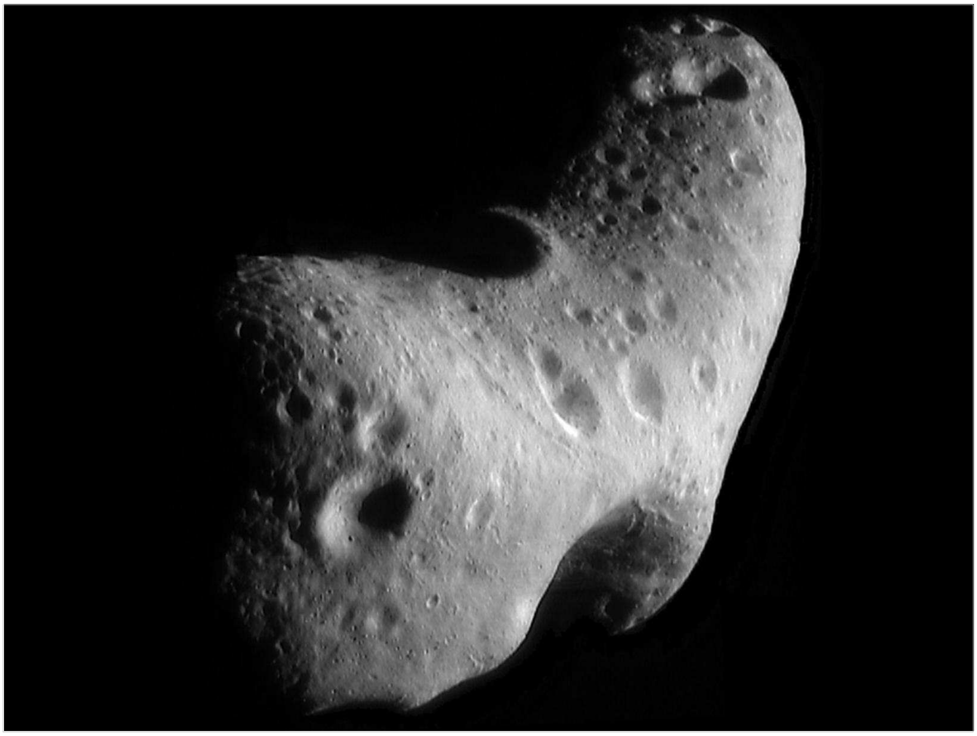
CASI ASTRO 2018

Who I am, what I'm talking about.

Two disclaimers: not an SFL project, and still work in progress – spare-time project and spare time has been short lately.



Near-Earth Asteroids of considerable interest for exploration, as hazards, as resources. Orbits vary a lot, significant e and i are common.

Delta-V an important criterion for choice, but hard to compute. Shoemaker & Helin, “Earth-Approaching Asteroids as Targets for Exploration”, 1978 paper: equations giving quick approximation, for screening. Many people have used the equations, but apparently nobody has taken a close look at them.

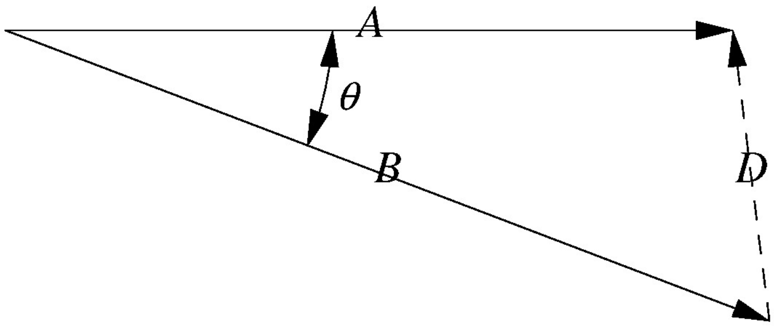
Student had a problem, couldn't get sensible results. That turned out to be a mundane units problem, but got me interested.

S&H were astronomers, not mission planners or orbital dynamics folks. How well did they understand their equations?

$$U_c^2 = \frac{3}{Q} - \frac{2}{Q+1} - \frac{2}{Q}\sqrt{\frac{2}{Q+1}} \cos\frac{i}{2}$$

The equations look rather strange, dimensionality weird and forms cryptic. Paper does not explain derivations. One reference, to an old Öpik paper: not too helpful, one equation for encounter velocity between Earth and an object in an elliptical orbit.

Part of the weirdness is the system of units. Earth orbit assumed circular with radius 1. Earth orbital velocity also taken as 1, implying solar GM = 1. Especially for a mission starting from Earth, permits many algebraic simplifications that normally aren't possible. Promiscuous simplifications produce the weirdness.



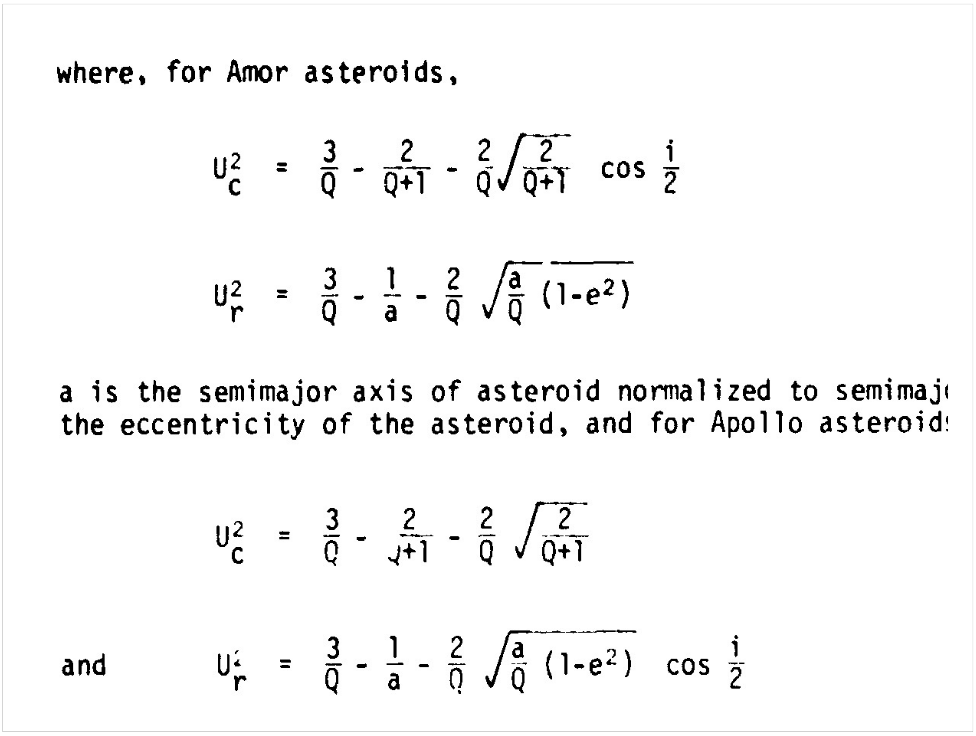
$$D^2 = A^2 + B^2 - 2AB\cos\theta$$

Some equations reminiscent of the cosine law: computes side of triangle given other sides and angle between them. Also gives magnitude of difference between two vectors, which is how S&H use it.

Given cosine law, a few bits of orbital dynamics, and some spherical trig for combining angles, only takes about half an hour to derive S&H equations, once you figure out their mission plan.

Actually can simplify some things from their forms – another hint that they didn't really understand the equations too well.

But their mission plan is weird.



where, for Amor asteroids,

$$U_c^2 = \frac{3}{Q} - \frac{2}{Q+1} - \frac{2}{Q}\sqrt{\frac{2}{Q+1}} \cos\frac{i}{2}$$

$$U_r^2 = \frac{3}{Q} - \frac{1}{a} - \frac{2}{Q}\sqrt{\frac{a}{Q}(1-e^2)}$$

a is the semimajor axis of asteroid normalized to semimaj
the eccentricity of the asteroid, and for Apollo asteroid

$$U_c^2 = \frac{3}{Q} - \frac{2}{Q+1} - \frac{2}{Q}\sqrt{\frac{2}{Q+1}}$$

and $$U_r^2 = \frac{3}{Q} - \frac{1}{a} - \frac{2}{Q}\sqrt{\frac{a}{Q}(1-e^2)} \cos\frac{i}{2}$$

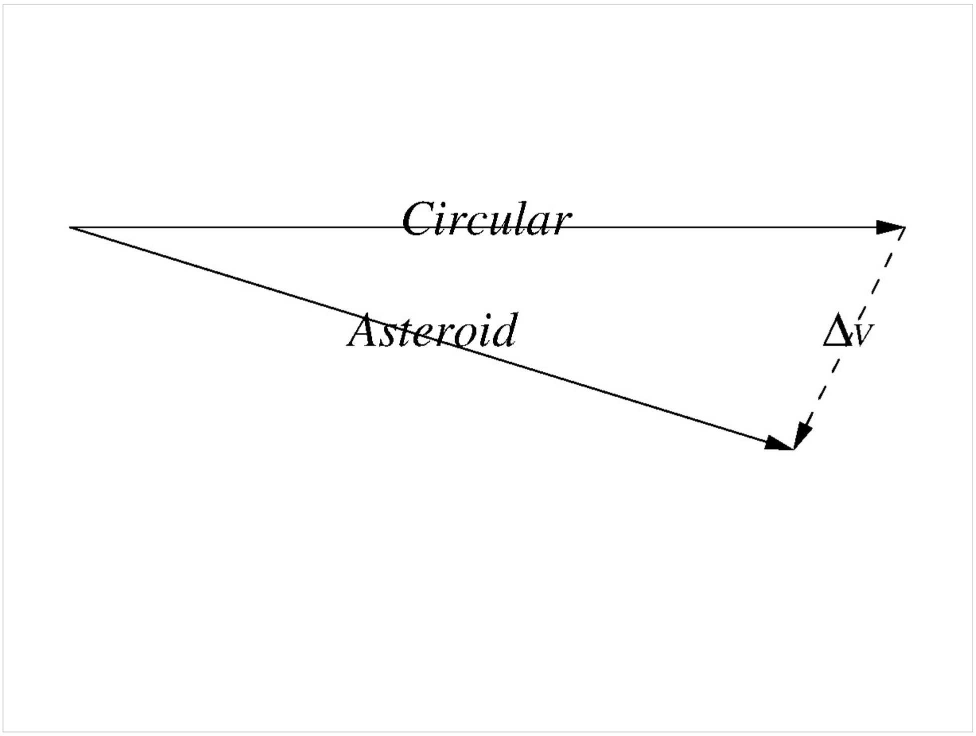
Okay, rendezvous mission using two burns, which is fine, with half the inclination change done in each, which is a reasonable approx for modest inclinations. The Earth departure maneuver is straightforward, and they meet the asteroid at aphelion (doesn't work for Atiras, but they were unknown then).

But the arrival maneuver is conceived as two parts: first circularize (!) at asteroid aphelion, then drop down into the asteroid's elliptical orbit, with the two parts then combined into a single burn. With two variations, one for Apollos and one for Amors, doing the remaining inclination change in part 1 or part 2. Repeatedly applying one equation they didn't really understand?

Fine if done right. But it's not!

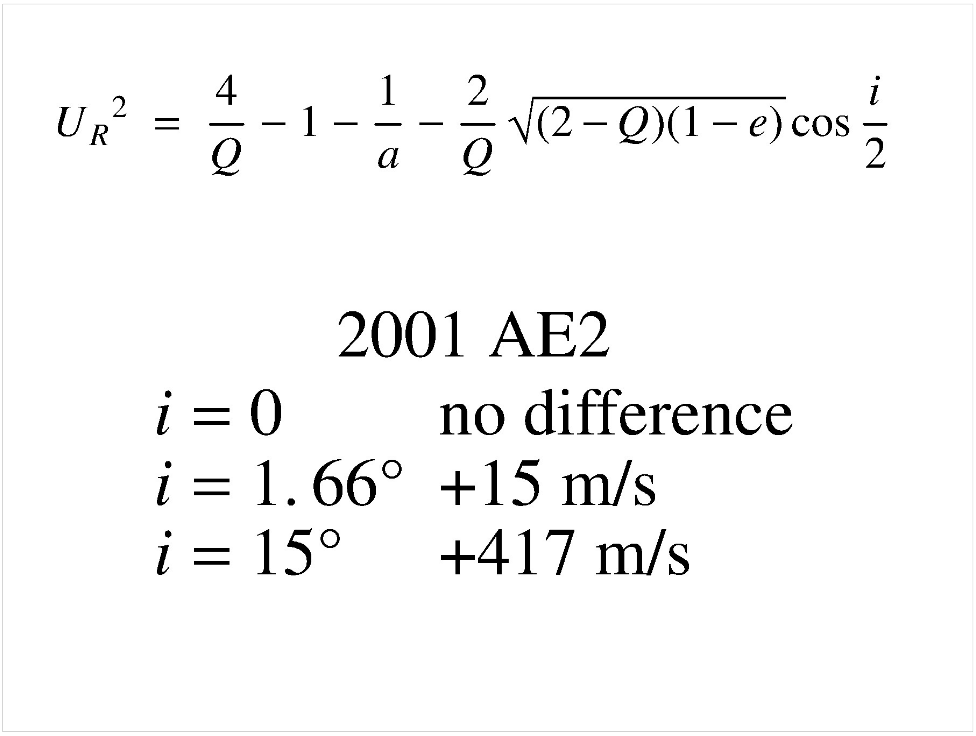
$$U_R = \sqrt{U_c^2 - 2U_rU_c \cos \frac{i}{2} + U_r^2}$$

They combine the two parts using the cosine law, which is fine. But it assumes that the angle between the two parts is the remaining inclination change, i/2… which is wrong.



Easiest case to understand is Apollos, with remaining inclination change done in the second part, dropping down into the asteroid orbit. This is the view from the Sun of the velocity vectors.

The first part, the circularization burn, is along the circular orbit's velocity vector. Break the drop burn into in-plane and out-of-plane components. Its in-plane component is difference between the in-plane components of the two vectors, but its out-of-plane component is the **whole** out-of-plane component of the new orbit. Result: drop burn's direction is nowhere near the asteroid's velocity vector, and hence is **not** at angle i/2 relative to the circular orbit.



$$U_R{}^2 = \frac{4}{Q} - 1 - \frac{1}{a} - \frac{2}{Q}\sqrt{(2-Q)(1-e)}\cos\frac{i}{2}$$

2001 AE2

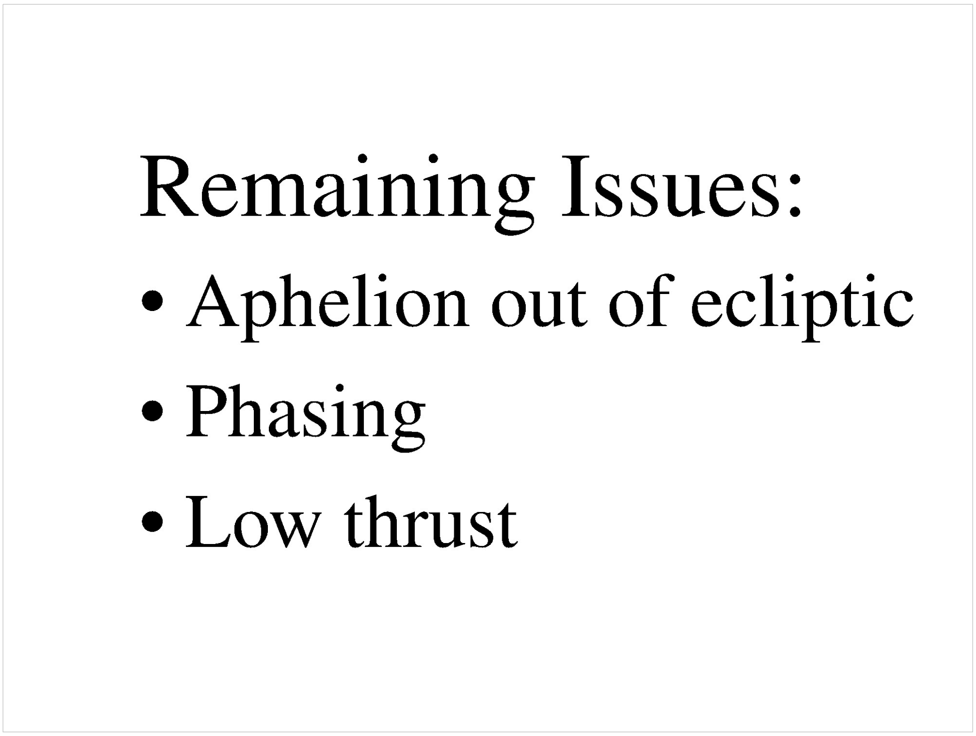
$i = 0$	no difference
$i = 1.66°$	+15 m/s
$i = 15°$	+417 m/s

The weirdest part is, it's not that hard to use the cosine law to go direct from the transfer orbit to the asteroid orbit, in one step, no combining needed, and it's right for all the asteroid classes. Took me five minutes to derive this. Why didn't they do this?

If inclination zero, makes no difference. Our standard example asteroid was 2001 AE2. For its actual inclination, about 1.7deg, difference slight.

But many NEAs quite inclined. Pretend 2001 AE2 is at 15deg, and S&H equations are nearly half a km/s low, quite significant. S&H's final step, empirically giving best agreement with optimized trajectories, is to add 500m/s fudge factor; could this be why?

They validated against a small set of asteroids; recent unpublished work with larger set shows poorer agreement – could this be why? Will investigate this.



Remaining Issues:

- Aphelion out of ecliptic
- Phasing
- Low thrust

Remaining issues…

S&H assume asteroid aphelion is in ecliptic. Hoping to generalize this, haven't managed it yet.

S&H ignore phasing, settle for reaching asteroid orbit without caring where asteroid will be at arrival time. Phasing penalties can be significant. Hard problem; I suspect no simple approximation possible.

S&H assume impulsive thrusting. Low thrust, e.g. electric, quite interesting. Difficult calculation, with major penalties possible in general, but for a not-too-elliptical orbit and a not-too-low thrust, first results suggest penalties might be surprisingly small. Needs confirmation.

And yes, need to write this up and publish it properly.

Questions?

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Any questions?