Orbit Design in the Context of the Circular Restricted Three-Body Problem: Application to Binary Asteroid Systems

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Introduction

The three-body problem has been studied since Euler and is one of the most exciting problems to work on in the field of spacecraft dynamics. The Circular Restricted Three-Body Problem (CRTBP) has been studied with space missions in mind intensively since the 1960s, leading to the discovery of families of orbits around the Lagrangian points in the Earth-Moon and Sun-Earth systems. Recently, orbits found in the context of the CRTBP have been used in mission such as Genesis, ISEE-E and Artemis. Now, researchers are pushing it a little further by designing orbital motion around binary small body systems, such as binary asteroids. The AIDA mission, planned to launch in 2020, will orbit the binary asteroid system Didymos (Cheng et al., 2013), enhancing the need to design orbital trajectories in its vicinity.

Binary small body systems such as Didymos are particular due to the fact that their bodies do not have a spherical shape. They also have low gravitational forces because of their small size. This means that perturbations like the Solar Radiation Pressure (SRP) and higher order gravitational potential can have a large impact on the orbital motion of a small spacecraft and must be included in the models used for orbit design (Jean et al., 2018).

As it is widely known, the orbits in the context of the CRTBP cannot be described with the familiar orbital elements used in a Keplerian orbit. Orbital motion must be found and designed using numerical techniques and a deep understanding of the dynamics of the system. The work described here is proposing a method that can be implemented to find periodic orbital motion in the context of the CRTBP. It also shows how to apply this method to binary small body systems, such as Didymos, using a 4^{th} order gravitational potential model, as well as the inclusion of the SRP.

The CRTBP

The CRTBP is the study of the dynamics of a system composed of three bodies: the primary bodies of a binary system and a spacecraft. The primary bodies of the binary system are both orbiting their barycenter in a circular fashion. The mass of the spacecraft is much smaller than the mass of the primary bodies, so it does not influence their motion.

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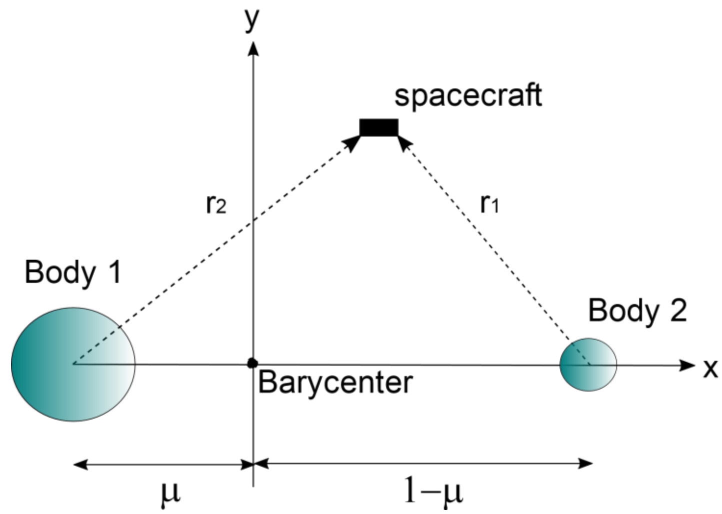

Figure 1- Description of the CRTBP

Equations of motion

The equations of motion are driven by the gravitational potential exerted by the primary bodies on the spacecraft and the rotational motion of the binary system. In the classical CRTBP, where the primary bodies are considered as point masses, the gravitational potential exerted by one of the primary body on a unit mass spacecraft is given by (Howell, 1984):

$$U_{grav} = \frac{(1-\mu)}{r_1} + \frac{\mu}{r_2}$$

Where μ is the mass ratio of the primary bodies of the binary system, and r_1 and r_2 are the distances between the corresponding primary body center of mass and the center of mass of the spacecraft.

Since the system is rotating around its barycenter, a pseudo-potential term, created by the centrifugal forces acting on the system is added to the total potential exerted on the spacecraft:

$$U = U_{grav} + \frac{1}{2}\Omega^2(x^2 + y^2)$$

Where Ω is the rotation rate of the binary system and x and y are the position of the spacecraft with respect to the barycenter in the synodic (rotating) reference frame. The final equations of motion for this system are given by:

$$\ddot{x} = 2\Omega\dot{y} + U_x$$
$$\ddot{y} = -2\Omega\dot{x} + U_y$$
$$\ddot{z} = U_z$$

Where U_x, U_x and U_x are the differentiation of U with respect to the x, y, and z components of the position vector of the spacecraft respectively. The $2\Omega\dot{y}$ and $-2\Omega\dot{x}$ terms are due to the Coriolis acceleration induced by the rotation of the binary system.

An integration constant, the Jacobi constant, can be extracted from the equations of motion. It is defined by the following expression:

$$C = 2U - (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$$

Adapting to non-spherical bodies

In the case of binary asteroid systems, the bodies composing them can not be considered as point mass as they have non-spherical shapes. The gravitational potential is then the sum of the gravitational potential of each mass element of the primary bodies. For one mass element, the distance between the mass element and the spacecraft is described with respect to the position of the center of mass of the body (Jean et al., 2018):

$$\boldsymbol{r} = \boldsymbol{R} - \boldsymbol{\delta}$$

Where $\boldsymbol{r}$ is the vector of the position of the spacecraft with respect to the mass element, $\boldsymbol{R}$ is the vector of the position of the spacecraft with respect to the center of mass of the primary body and $\boldsymbol{\delta}$ is the vector of the position of the mass element with respect to the center of mass of the primary body.

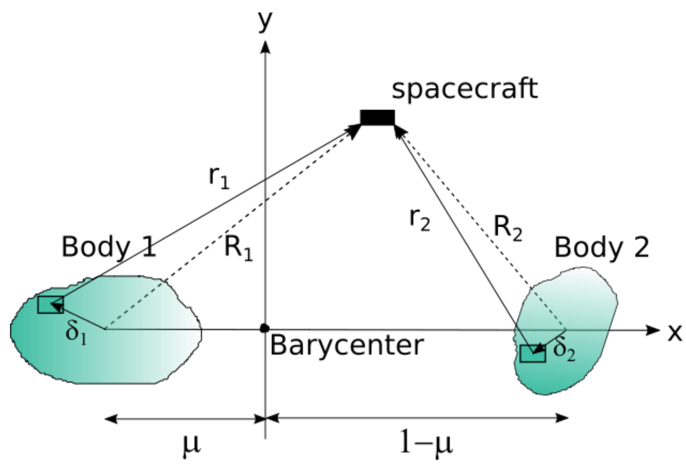

Figure 2- Geometry of the CRTBP for non-spherical bodies

For one mass element, the gravitational potential is then:

$$U = \frac{Gdm}{|\boldsymbol{R} - \boldsymbol{\delta}|}$$

The total gravitational potential of the binary system is calculated by integrating this expression over the entire mass of both primary bodies composing it:

$$U_{grav} = \int \left(\frac{G}{|\boldsymbol{R}_1 - \boldsymbol{\delta}_1|}\right) dm_{a1} + \int \left(\frac{G}{|\boldsymbol{R}_2 - \boldsymbol{\delta}_2|}\right) dm_{a2}$$

To be able to do this, the terms to be integrated have been expanded through a Taylor series. Terms until the fourth order were kept for the integration. The total gravitational potential for the binary system is calculated by integrating the expansion terms over the entire mass of the body.

Search for periodic orbits for a 0th order gravitational potential model

The search for periodic orbits is done through a continuation scheme where the equations of motions are integrated for different initial conditions. For one particular initial position of the spacecraft, a range of Jacobi constant is used, chosen based on the Jacobi constants of the equilibrium points. For each Jacobi constant, an initial velocity is calculated, and the equations of motion are solved numerically until the spacecraft crosses the X-axis twice. The final and initial states of the system are then compared. A plot of the difference between the initial and final position is created. When the plot crosses zero, a candidate periodic is found. The crossings that are of interest for periodic orbits are the ones where the system goes from being totally unpredictable to a smoother curve or vice-versa. These transitions denote a change in the dynamics of the system and are signs of possible periodic motion. In the example plot of Figure 3, there are two C values for candidate periodic orbits are: 2.9048 and 3.1005.

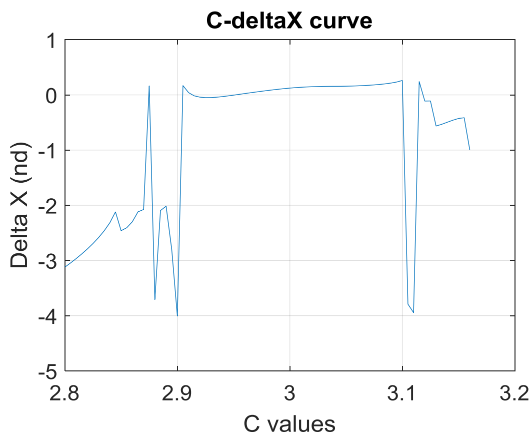

Figure 3- Plot of the difference between initial and final position

The corresponding initial conditions are tested to know if the resulting trajectory is indeed periodic. When it is close to periodic, it is then adjusted using a differential-correction scheme, as described in (Heiligers et al., 2016).

Higher order gravitational potential model

Once a suitable set of initial conditions has been found for a 0th gravitational potential model, it is possible to modify it by using a combination of a continuation scheme and a differential-correction scheme. To do so, the gravitational potential is divided into three parts: one for the 0th order terms, one for the 2nd order terms and one for the 4th order terms. The 2nd and 4th order terms are assigned a 2nd order and 4th order factor, respectively:

$$U = U_0 + U_2 f_2 + U_4 f_4$$

The f_2 and f_4 factors are first slightly increased, then the differential-correction scheme is applied to find the initial conditions which would produce a periodic trajectory in the corresponding model. Once f_2 and f_4 are both equal to 1, the search for a periodic orbit based on the 4th order gravitational potential model is completed.

Solar radiation pressure

The solar radiation pressure is obtained using a flat plate model, where the force due to the SRP from a distant point source is (McInnes, 2004):

$$\boldsymbol{F}_{SRP} = M_{sc} \frac{L(1+\rho)}{4\pi c b^2 B} (\hat{\boldsymbol{u}} \cdot \hat{\boldsymbol{n}})^2 \hat{\boldsymbol{n}}$$

where ρ is the reflectivity parameter of the spacecraft, B is the spacecraft mass to area ration, c is the speed of light, b is the distance of the spacecraft from the Sun, L is the solar luminosity, $\hat{\boldsymbol{u}}$ is the unit vector from the Sun to the spacecraft and $\hat{\boldsymbol{n}}$ is the unit vector normal to the spacecraft surface exposed to the Sun.

To be able to design closed-form trajectories including the SRP, the period of the orbits chosen have to be a fraction of the period of the binary system. A continuation scheme is applied to a nominal orbit to make it compatible to a model that includes the SRP.

Results

This method has been applied to planar orbits, such as planar Lyapunov and Distant Retrograde Orbits (DROs) to a system representing the binary asteroid system Didymos (Dell'Elce et al., 2017). Families of orbits have been found in the 0th gravitational potential model. Examples of planar Lyapunov families and DRO families are shown in Figure 4. In the figure, the black ellipse is the secondary body of the binary asteroid system. The primary body of the binary asteroid system is not shown. The red orbits, represent the orbits that have the right period for the SRP. They will be modified so that they exist the 4th order gravitational potential model and the SRP.

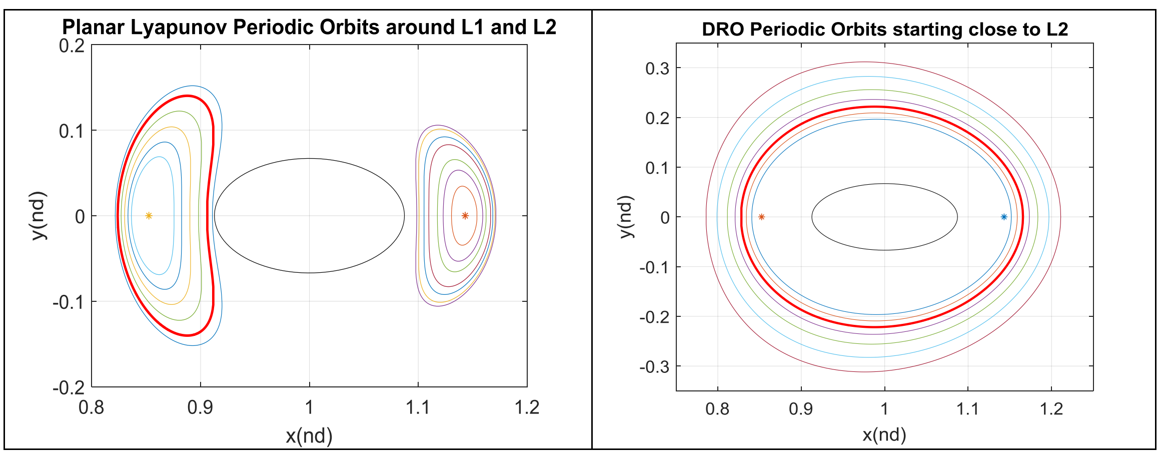

Figure 4- Families of Lyapunov and DRO orbits, 0th order gravitational potential

An example of periodic orbits that have been modified, so that they exists in the 4th order gravitational potential model, are shown in Figure 5 for both Lyapunov orbits and DROs.

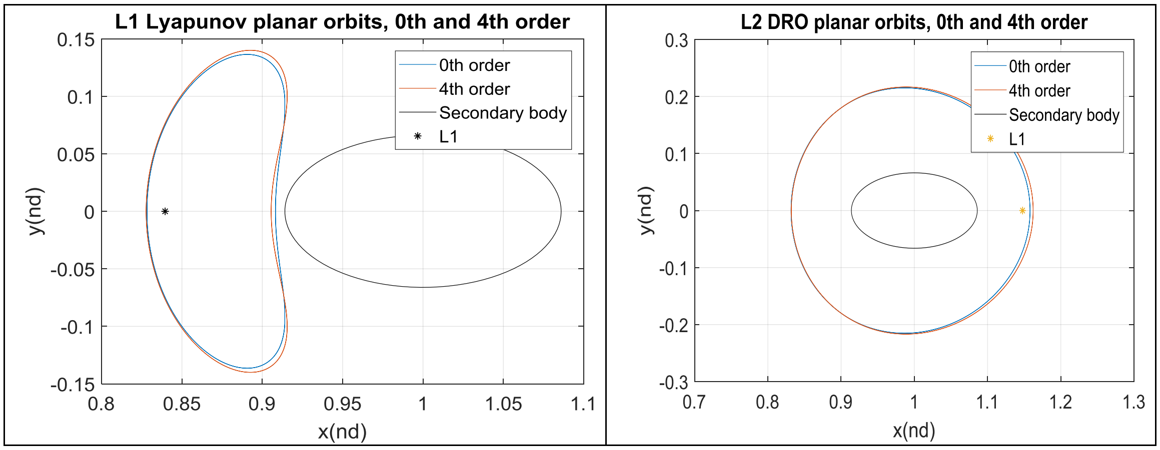

Figure 5- Orbits using 0th order and 4th order gravitational potential

When the SRP is added into the model, the trajectories have the following behavior shown in Figure 6. In the actual case, the chosen orbits have a period that is half the one of the system's period. Because of that, it takes two rotations before the spacecraft goes back to its initial conditions. The trajectory then closes after two orbits. In Figure 6, the left figure (Lyapunov trajectory) is zoomed to show the trajectory detail.

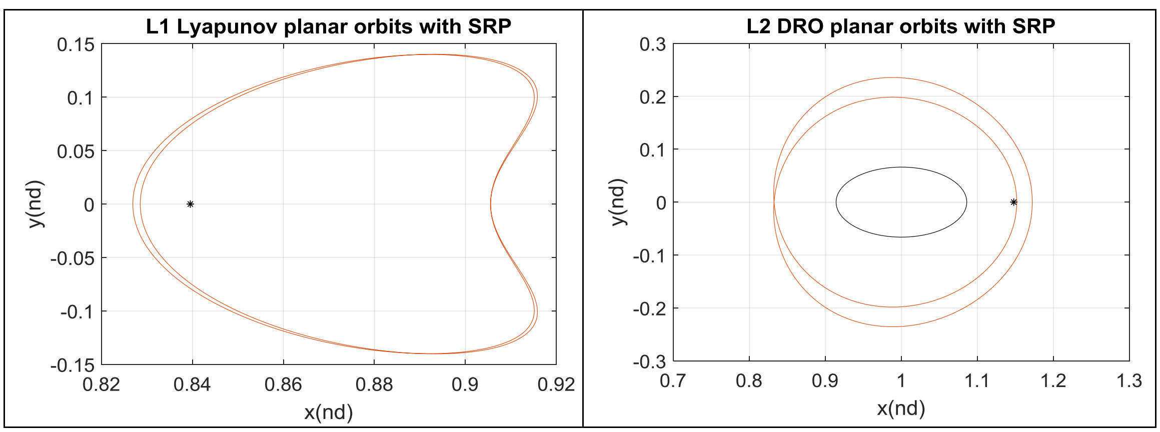

Figure 6- Trajectories including SRP and 4th order gravitational potential

Conclusions and future work

A method that can be used to find planar periodic orbits in the CRTBP, applied to binary asteroid, system has been fully described. It showed that it requires good knowledge of the complex dynamics of that type of systems. Special numerical techniques were required, which permitted to compute planar Lyapunov and DRO orbital trajectories. Perturbations, such as the SRP also play an important role in the dynamics of a spacecraft in the vicinity of binary asteroids. Future work on the subject will include a study on the possibility to use it to do orbit control and maintenance.

References

Cheng, A.F., Rivkin, A.S., Reed, C., Barnouin, O., Fletcher, Z., Ernst C., AIDA: Asteroid Impact and Deflection Assessment, 64th International Astronautical Congress, Beijing, China, IAC-13-A3.4.8, September 2013

Dell'Elce, L., Baresi, N. Naidu, S.P., Benner, L.A.M, Scheeres, D.J., Numerical investigation of the dynamical environment of 65803 Didymos, Advances in Space Research, Vol. 59, 2017, pp. 1304-1320

Heiligers, J., Macdonald, M., Parker, J.S., Extension of Earth-Moon libration point orbits with solar sail propulsion, Astrophysics and Space Science, Vol 361, Issue 7, 2016

Howell, C.K., Three-Dimensional, Periodic, 'Halo' Orbits, Celestial Mechanics, Vol.32, Issue1, 1984, pp. 53-71

Jean, I., Misra, A.K., Ng, A., Dutta, S., Orbital dynamics of a spacecraft in the vicinity of a binary asteroid system: Impact of solar radiation pressure on orbital motion, 2018 Space Flight Mechanics Meeting, AIAA SciTech Forum, Kissimmee, Florida, AIAA 2018-0956, January 2018

McInnes, C.R., Solar Sailing: Technology, Dynamics and Mission Applications, Springer-Verlag Berlin Heidelberg, 2004