A performance prediction method for Aero-Naut CAM folding propellers is investigated through parametric studies of existing wind tunnel testing data. Based on propeller’s pitch-to-diameter ratios (P/D), thrust and power coefficient for varying advance ratios are estimated using 4th order polynomial curve fitting, from which a propeller efficiency is determined. A motor efficiency performance is estimated using 4th order surface fit as a function of motor RPM and propeller torque from test results conducted at Ryerson Applied Aerodynamics Laboratory of Flight (RAALF). Using operating flight conditions of CREATeV solar aircraft, an iterative method is developed where propeller thrust and power coefficients, torque, RPM, as well as propeller and overall efficiency at various P/D ratios are computed using modified Newton-Raphson method. The resultant analysis shows optimum P/D ratio of 0.846 with propeller diameter of 17 inches. Compared to the previous 18X9 Aero-Naut CAM folding propeller, it is found that optimized propeller selections would increase overall efficiency by 3–4%. While the magnitude of efficiency increase seems marginal, the performance gain of CREATeV’s ultra-long-range capability is expected to be significant.

Nomenclature

CREATeV = Clean Renewable Energy Aerial Test Vehicle
RAALF = Ryerson Applied Aerodynamics Laboratory of Flight
RPM = revolutions per minute
SL = sea level
UAV = unmanned aerial vehicle
UIUC = University of Illinois at Urbana – Champaign

\( C_P \) = propeller power coefficient
\( C_T \) = propeller thrust coefficient
\( D \) = propeller diameter
\( h_p \) = density altitude
\( J \) = advance ratio
\( n \) = revolution per second
\( n_{rpm} \) = revolution per minute

\( p \) = pressure
\( \mathcal{P} \) = propeller power
\( \text{P/D} \) = pitch-to-diameter ratio
\( Q \) = propeller torque
\( R \) = universal gas constant
\( T \) = temperature
\( \mathcal{T} \) = propeller thrust
\( V \) = freestream velocity

\( \eta_{motor} \) = motor efficiency
\( \eta_{prop} \) = propeller efficiency
\( \eta_t \) = overall efficiency

\( \rho \) = density

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I. Introduction

Ryerson University’s Clean Renewable Energy Aerial Test Vehicle (CREATeV), shown in Fig. 1, is a 12.5kg ultra-long-endurance, solar-powered unmanned composite aircraft with the goal of continuous sustained flight endurance of 60 days. Along the upper surface of 6.28-metre-long wing lies 96 solar cells which would recharge the battery during hours of daylight and would sustain the flight during nightfall for maximum of 12 hours [1]. Due to the unique mission profile of CREATeV of maintaining optimal flight condition during its ultra-long endurance run, a greater degree of propeller-motor optimization was required. Dantsker et al. [2] estimated that about 52% of the total electrical power consumption of computationally intensive small unmanned aerial vehicles (UAVs) is caused by propulsion. Therefore, there exists a significant potential for performance improvement from achieving greater overall propulsion efficiency, benefitting wide variety of high-performance and general-use UAVs as well.

The aircraft currently utilizes selections of 18 or 20-inch Aero-Naut CAM folding propellers with T-MOTOR U7-V2.0 280KV electric motor, selected based on wind tunnel testing data conducted at Ryerson Applied Aerodynamics Laboratory of Flight (RAALF). Examples of Aero-Naut propeller blades can be seen in Fig. 2. Although wide range of Aero-Naut CAM folding propellers were recently tested at University of Illinois at Urbana-Champaign (UIUC) by Dantsker et al. [4], no published propeller test data was found for diameter greater than 16 inches. While independent wind tunnel testing remains a viable and reliable option for obtaining propeller performance results, propeller optimization process would require numerous diameters and pitch geometries to be tested for proper performance comparison. Such endeavour quickly becomes too cost-intensive due to time-consuming and labour-intensive nature of conducting wind tunnel testing in addition to the expensive wind tunnel machinery operating costs.

More cost-effective, rapid, and relatively accurate analysis methods have been proven for different propeller models where Jessa [5] determined low-Reynolds-number scaling relations of aerodynamics and propeller performance for varying rotor diameter of T-MOTOR carbon fiber rotors. For an identical pitch geometry of different diameters, there existed a near-identical chord and twist distributions and near-identical aerodynamics and performance coefficients.

Therefore, a research was conducted for the analysis into a scaling relation of Aero-Naut CAM folding propeller performance through parametric studies of published wind tunnel data. This paper describes the methodology for identifying scaling relations for Aero-Naut CAM folding propellers, propeller performance prediction iteration method and its results as well as important summary and its application into CREATeV.
II. Methodology

A. Propeller Scaling Relations

In order to accurately predict thrust and power coefficients of Aero-Naut folding propellers with various size and geometry, a performance scaling relationship based on propeller geometric parameters must be identified and validated. As an example, Jessa [5] identified performance scaling relations of T-motor rotors with identical propeller pitch geometries. For Aero-Naut folding propellers, Dantsker et al. [4] identified a trend where thrust, power, and efficiency coefficients depend on P/D ratio, suggesting a possible existence of performance scaling relationship with propeller geometry.

However, difficulties in determining the performance scaling relationship for Aero-Naut CAM folding propellers were also identified. Dantsker et al. [4] had investigated geometric characteristics of the folding propellers of various diameter and pitch sizes, of which they discovered that for varying pitch number and P/D ratio for given diameter, the propeller blade geometry was different and unique from one another.

Therefore, the level of validity and applicability of performance scaling relations for different propeller models had to be understood and demonstrated. To do so, effects of propeller pitch number and propeller P/D ratio on performance coefficients were investigated when analyzing performance scaling relationship for Aero-Naut CAM folding propellers. For compiling propeller coefficient values in determining performance scaling relationship, wind tunnel test results recorded at UIUC was utilized due to the availability for wide variance of propeller size and pitch [4]. Once the characteristic parameter was determined, the corresponding thrust and power coefficient functions were estimated by plotting series of appropriate propeller data of varying diameter, then extrapolating equation of 2nd to 4th order using polyfit function on MATLAB.

B. Propeller Coefficients

If air is assumed to behave as an ideal gas, and if two parameters are known from temperature, pressure, and density, ideal gas law can be used to determine atmospheric properties of interest. The ideal gas law is defined as,

\[ p = \rho RT \]  

(1)

Additionally, if target flight altitude is known, density of operating condition could be estimated from density altitude definition provided by Gudmundsson [7], for altitudes below 36.089ft (11,000m) and provided in feet.

\[ \rho = \rho_{SL} \left[ 1 - \frac{h_p}{145442} \right]^{\frac{1}{0.234957}} \]  

(2)

Propeller power is determined by propeller torque and its angular velocity:

\[ P = 2\pi nQ \]  

(3)

Non-dimensionalized power and thrust coefficients, respectively, are determined using,

\[ C_p = \frac{P}{\rho n^3 D^5} \]  

(4a)

\[ C_T = \frac{T}{\rho n^2 D^4} \]  

(4b)

Freestream and rotational velocity terms can be non-dimensionalized in terms of advance ratio, \( J \), and is determined as,

\[ J = \frac{V}{nD} \]  

(5)
The efficiency of the propeller is defined as,

\[ \eta_{prop} = \frac{T_V}{P} = \frac{C_T}{C_P} \quad (6) \]

If motor efficiency at operating condition is known, the overall efficiency at operating condition is defined as,

\[ \eta_t = \eta_{motor}\eta_{prop} \quad (7) \]

Where \( \eta_{motor} \) denotes the efficiency of the motor.

C. Motor Performance Prediction

The efficiency curve of T-MOTOR U7-V2.0 280KV motor for CREATeV was obtained by analyzing wind tunnel testing data of various propellers conducted at RAALF. A surface fit estimation of motor efficiency was constructed via MATLAB using propeller torque and motor RPM of test conditions, and its plot can be seen in Fig. 3. It was determined that the 4th-order approximation would result in accurate surface fit model at its expected operating test conditions.

Figure 3: Surface fit implementation for T-MOTOR U7-V2.0 motor efficiency.
D. CREATeV Propulsion Performance Prediction

<table>
<thead>
<tr>
<th>Target Operating Condition of CREATeV.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Thrust ((T)) [N]</strong></td>
</tr>
<tr>
<td><strong>Airspeed (V) (\text{[m/s]})</strong></td>
</tr>
<tr>
<td><strong>Density Altitude (h_p) [ft]</strong></td>
</tr>
</tbody>
</table>

Target optimum thrust and freestream velocity, and its target density altitude of CREATeV are expected to fall within a range given in Table 1. Given its target operating conditions and diameter, Equation (4b) can be rearranged into homogenous polynomial equation function of \(n\) and be solved for roots. For a relative rapid convergence for the iteration simplicity, a modified Newton-Raphson method suggested by McDougall and Wotherspoon [8] was utilized:

\[
\begin{align*}
  x_0^* &= x_0 \\
  x_1 &= x_0 - \frac{f(x_0)}{f'(x_0)} \\
  x_k^* &= x_k - \frac{f(x_k)}{f'(x_k)} \quad (8a) \\
  x_{k+1} &= x_k - \frac{f(x_k)}{f'(x_k)} \quad (8b)
\end{align*}
\]

For \(k \geq 1\),

\[
\begin{align*}
  x_k^* &= x_k - \frac{f(x_k)}{f'(x_k)} \quad (9a) \\
  x_{k+1} &= x_k - \frac{f(x_k)}{f'(x_k)} \quad (9b)
\end{align*}
\]

Where, if \(x=n\),

\[
\begin{align*}
  f(n) &= (\rho D^4 C_T) n^2 - T \quad (10a) \\
  f'(n) &= 2(\rho D^4 C_T) n \quad (10b)
\end{align*}
\]

Once value of \(n\) is obtained through modified Newton-Raphson method, power coefficient and advance ratio of the propeller were estimated using Equations (4a) and (5), respectively. Its propeller efficiency was calculated using Equation (6), along with expected propeller torque through Equation (3).

Overall propulsive efficiency of CREATeV is a simple product of propeller and motor efficiency as per Equation (7). Its maximum overall efficiency can easily be isolated and be plotted for varying diameters and can be iterated for varying range of characteristic geometry parameters. A simplified representation of the propeller-motor performance prediction is shown in Fig. 4.
III. Results

Test data of Aero-Naut CAM folding propellers published by Dantsker et al. [4] with diameters ranging from 10 to 16 inches were used for the analysis. Parametric analysis in relations to propeller pitch number yielded relative wide discrepancy in propeller efficiency as well as thrust and power coefficients, which is presented in Fig. 5 and 6, respectively, with estimated curve fit is shown in solid black plot. The results are in line with Aero-Naut propellers’ unique geometric profiles, suggesting that relatively poor performance scaling relations exist based on propeller pitch number for Aero-Naut CAM folding propellers.

Results of parametric analysis in relations to P/D ratio showed greater degree of conformity for propeller efficiency and thrust and power coefficients, as shown in Fig. 7 and 8, respectively, with average P/D ratios provided in bracket for given plot colours. The degree of conformity varied depending on groups of similar P/D ratios, with propellers whose average P/D ratio of 0.846 showed the widest discrepancy of thrust and power coefficients. As mentioned, such discrepancies are in line with unique geometric profiles of each Aero-Naut propellers. For propeller efficiency curves based on Fig. 7, consistency in efficiency values were observed for groups of similar P/D ratios, and accurate propeller efficiency estimation is expected from this performance scaling relations. Therefore, it was determined that Aero-Naut CAM folding propeller performance can be estimated with good accuracy based on scaling relations using similar P/D ratios.

Estimated thrust and power coefficient curve fit are presented in Appendix A, and performance graphs of propellers categorized by similar P/D ratios are presented in Appendix B.
Figure 5: Experimental thrust and power coefficient curves with 8-inch pitch for varying diameters.

Figure 6: Experimental propeller efficiency curves of propellers with varying P/D ratio.
Figure 7: Experimental thrust coefficient curves of propellers with varying P/D ratio.

Figure 8: Experimental power coefficient curves of propellers with varying P/D ratio.
Figure 9: Propeller efficiency curves of propellers with varying P/D ratio.

Performance prediction results for maximum attainable propeller and overall efficiencies for varying propeller geometry can be seen in Fig. 10 and 11, respectively. For the optimum operating condition of CREATEv, it was found that propellers with P/D ratio of 0.846 had the highest propeller and overall theoretical efficiencies with values of 0.7919 and 0.6711, respectively.

Figure 10: Theoretical maximum propeller efficiency for varying propeller geometry.
Figure 11: Theoretical maximum overall efficiency for varying propeller geometry.

Several important analysis outcomes regarding efficiency optimization had been observed. Firstly, an increase of P/D ratio of 0.846 and thereafter did not result in greater performance gain but caused significant efficiency degradation. Secondly, an increase of propeller diameter did not guarantee an efficiency increase, and there existed the optimum propeller diameter for given P/D ratio that provided maximum of maximum attainable overall efficiencies. Lists of theoretical optimum propellers and their operating conditions can be found in Table 2. Several Aero-Naut CAM folding propellers found in catalogues were recommended based on performance prediction results and can be found in Table 3. It should also be noted that due to lack of available propellers as P/D ratio increases, only one or two propellers analysis were available for P/D ratio greater than 0.8, with the only exception being P/D ratio of 0.846. This however was not expected to severely affect the accuracy of propeller performance, as the performance scaling effect based on P/D ratio had been demonstrated as per Fig. 7 through 9.

Table 2: Theoretical Optimum Propellers for Maximum Efficiency at Operating Condition of CREATeV

<table>
<thead>
<tr>
<th></th>
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<tbody>
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<td>0.4</td>
<td>18.50</td>
<td>7.40</td>
<td>0.6130</td>
<td>11.0</td>
<td>7.0</td>
<td>125.51</td>
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<td>0.5</td>
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<td>0.6</td>
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<td>10.80</td>
<td>0.6475</td>
<td>11.0</td>
<td>5.0</td>
<td>84.88</td>
<td>0.258</td>
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<td>0.7</td>
<td>17.50</td>
<td>12.25</td>
<td>0.6446</td>
<td>11.0</td>
<td>5.0</td>
<td>85.27</td>
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<td>0.8</td>
<td>17.00</td>
<td>13.60</td>
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<td>11.0</td>
<td>5.0</td>
<td>83.43</td>
<td>0.274</td>
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<tr>
<td>0.846</td>
<td>17.00</td>
<td>14.38</td>
<td>0.6711</td>
<td>11.0</td>
<td>5.0</td>
<td>81.89</td>
<td>0.269</td>
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<td>0.9</td>
<td>16.50</td>
<td>14.85</td>
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<td>5.0</td>
<td>84.51</td>
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<tr>
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<td>5.0</td>
<td>95.42</td>
<td>0.359</td>
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Table 3: Recommended Aero-Naut CAM propellers for CREATeV

<table>
<thead>
<tr>
<th>Propeller</th>
<th>P/D</th>
<th>Estimated Overall Efficiency</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>18X9</td>
<td>0.50</td>
<td>0.6288</td>
<td></td>
</tr>
<tr>
<td>20X10</td>
<td>0.50</td>
<td>0.6217</td>
<td>Current</td>
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<tr>
<td>16X13</td>
<td>0.81</td>
<td>0.6575</td>
<td></td>
</tr>
<tr>
<td>14X12</td>
<td>0.86</td>
<td>0.6512</td>
<td>Recommended</td>
</tr>
<tr>
<td>18X11</td>
<td>0.61</td>
<td>0.6475</td>
<td></td>
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<tr>
<td>17X11</td>
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<td>0.6456</td>
<td></td>
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<tr>
<td>15X13</td>
<td>0.87</td>
<td>0.6440</td>
<td></td>
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<tr>
<td>13X11</td>
<td>0.85</td>
<td>0.6347</td>
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</table>

Although maximum of 3 to 4% overall efficiency increase is anticipated when compared to current propellers, it is expected to enhance ultra-long-endurance capability of CREATeV and greatly benefit from its increased mission capability.

IV. Conclusion

This paper investigated performance prediction method for Aero-Naut CAM folding propellers through parametric studies of existing wind tunnel testing data. The main objective was to optimize propeller selection of CREATeV using the estimated performance scaling relations of the propeller geometry determined from parametric studies. Due to the unique geometric profiles of every Aero-Naut folding propellers, it was proven to be challenging in determining proper performance scaling effect through parametric studies. Nevertheless, it was found that scaling relations in relations to propeller P/D ratio provided relatively accurate performance estimates, especially the propeller efficiency. Further analysis into the efficiency at CREATeV’s operating condition showed optimized P/D ratio of 0.846 with theoretical optimum propeller diameter of 17 inches. Compared to the current selection of 18X9 Aero-Naut CAM propeller, the efficiency increase is estimated to be about 4%. When selecting the existing model of 16X13, the efficiency increase is about 2 to 3%. While the magnitude of efficiency increase is marginal, the benefit towards ultra-long-endurance mission requirement of CREATeV is expected to be significant.

For future work, performance of candidate propellers would be validated through wind tunnel testing conducted at RAALF. Other works may include in-depth measurements of Aero-Naut CAM propellers of interests for geometric characteristics such as propeller twist angle. Finally, performance results during full-scale flight testing of CREATeV may provide realistic impact on performance prediction results presented in this paper.
## Appendix A: Thrust and Power Coefficient Curve Fit Equations

### Table 4: Thrust Coefficient Curve Fit

<table>
<thead>
<tr>
<th>$P/D$</th>
<th>$J^4$</th>
<th>$J^3$</th>
<th>$J^2$</th>
<th>$J^1$</th>
<th>$J^0$</th>
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### Table 5: Power Coefficient Curve Fit

<table>
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<tr>
<th>$P/D$</th>
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<th>$J^3$</th>
<th>$J^2$</th>
<th>$J^1$</th>
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Appendix B: Performance Graphs of Analyzed Propellers (Test Results from UIUC [4])

Figure 12: (a) propeller efficiency, (b) thrust coefficient, and (c) power coefficient of propeller with \( P/D = 0.4 \).
Figure 13: (a) propeller efficiency, (b) thrust coefficient, and (c) power coefficient of propeller with P/D = 0.5.
Figure 14: (a) propeller efficiency, (b) thrust coefficient, and (c) power coefficient of propeller with P/D = 0.6.
Figure 15: (a) propeller efficiency, (b) thrust coefficient, and (c) power coefficient of propeller with P/D = 0.7.
Figure 16: (a) propeller efficiency, (b) thrust coefficient, and (c) power coefficient of propeller with P/D = 0.8.
Figure 17: (a) propeller efficiency, (b) thrust coefficient, and (c) power coefficient of propeller with $P/D = 0.846$. 
Figure 18: (a) propeller efficiency, (b) thrust coefficient, and (c) power coefficient of propeller with P/D = 0.9.
Figure 19: (a) propeller efficiency, (b) thrust coefficient, and (c) power coefficient of propeller with P/D = 1.1.
Figure 20: (a) propeller efficiency, (b) thrust coefficient, and (c) power coefficient of propeller with P/D = 1.2.
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