Aerodynamic Optimization of Unsteady Chaotic Flows

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1. Introduction

Within the next decade, global demand will necessitate more commercial aircraft, leading to an increase in Carbon Dioxide emissions using the current aircraft designs and technologies [1]. To avoid this, a $20 - 25\%$ reduction in fuel consumption is required to meet the current, and future, emissions regulations [1]. This goal highly depends on future aerodynamic designs, such as low-drag fuselages, high-lift wings, and more efficient jet engines. Therefore, improving aerodynamic optimization techniques for next-generation aircraft is a primary challenge in the aerospace community [2]. To achieve this goal, aircraft performance should be improved by accurate designs, and this accuracy can be obtained by high-fidelity aerodynamic optimization using scale-resolving solvers, such as Large-Eddy Simulation (LES) and Direct Numerical Simulation (DNS). These high-fidelity solvers provide more details, such as the effect of unsteady and turbulent flows on the designs. Therefore, with the advent of powerful computational tools, these solvers are highly demanded for future aerodynamic designs. However, solving optimization problems has historically been difficult, or impossible, for turbulent and chaotic fluid physics, where chaotic behaviour of non-linear terms in the Navier-Strokes equations contaminate the sensitivity functions, which yields impractical solutions in the design space. Shape optimization of turbulent flows is particularly challenging, since they induce

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chaotic divergence of sensitivity function, precluding the use of standard optimization approaches [3]. The present study aims to solve this issue and apply multi-disciplinary optimization for highly non-linear systems in the aerodynamic application.

2. Methodology

The present work proposes a method for solving high-fidelity and large-scale chaotic optimization problems. We convert PDE-constrained optimization into a Reduced-Order Model (ROM)-constrained form, where unstable modes are controlled to reduce computational cost. The minimization problem for this class of optimization is defined as

$$
\begin{aligned}
&\underset{\tilde{\boldsymbol{q}}\in\mathbb{R}^r, \mathcal{S}\in\mathscr{D}}{\text{minimize}} \quad \overline{\mathcal{J}} = \frac{1}{T}\int_0^T \mathcal{J}(\overline{\boldsymbol{u}} + \tilde{\boldsymbol{\Phi}}\tilde{\boldsymbol{q}}, t, \mathcal{S})dt, \\
&\text{subject to} \\
&\qquad \mathcal{R}(\overline{\boldsymbol{u}} + \tilde{\boldsymbol{\Phi}}\tilde{\boldsymbol{q}}, t, \mathcal{S}) = 0, \\
&\qquad \frac{d\mathcal{R}}{d\mathcal{S}}(\overline{\boldsymbol{u}} + \tilde{\boldsymbol{\Phi}}\tilde{\boldsymbol{q}}, t, \mathcal{S}) = 0, \\
&\qquad \mathbf{C}(\overline{\boldsymbol{u}} + \tilde{\boldsymbol{\Phi}}\tilde{\boldsymbol{q}}, \mathcal{S}) \leq 0,
\end{aligned}
\tag{1}
$$

where, $\overline{\boldsymbol{u}}$ is reference vector for states, $\tilde{\boldsymbol{\Phi}}$ is the trial basis function, $\tilde{\boldsymbol{q}}$ represents the generalized coordinates, $\mathcal{J}$ is the objective function. Moreover, $\mathcal{R}$ is the residual of the ROM, which is the reduced model of the high-dimensional governing equations, and $\mathbf{C}$ represents a constraint function. The significant achievement of the present approach is that solving large-scale optimization is now feasible for strongly non-linear problems. To this end, the Navier-Strokes equations are converted into the weak form using the Petrov-Galerkin approach in order to obtain a closure model in the form of a ROM. This closure model has lower dimensions, and its dynamical evolution embeds in a non-physical space (i.e., Hilbert space). Transformation of the governing equations into this Hilbert space allows us to apply constraints on the shape of the solution manifolds to develop accurate ROMs.

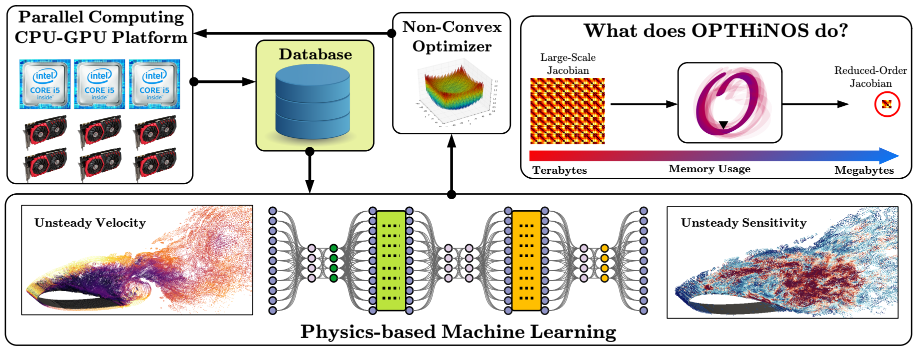

Figure 1: Schematic of high-fidelity optimization using present approach, and how the OPTHiNOS can help compute the sensitivities.

3. Physics-based machine learning

A physics-based machine learning approach is developed to build the manifolds in Hilbert space. Sequential optimization problems are also devised in the form of a feed-forward network. Then, the closure model is trained using data collection to reshape manifolds in Hilbert space. Unlike conventional approaches, where a black-box model is developed by processing massive datasets, the present machine learning framework produces an interpretable model that can be used for sensitivity analysis. In other words, this approach is able to reduce the size of high-dimensional Jacobian matrices significantly, while the lower-dimensional matrices still contain the primary physics of the problem. To compute the sensitivity solutions in this low-dimensional space, the shadowing lemma in the form of Least-Squares Shadowing (LSS) minimization is used. It is worth mentioning that conventional LSS is prohibitively expensive for sensitivity analysis of high-dimensional models. Hence, the objective of the present approach is to apply LSS to an accurate and interpretable ROM. Subsequently, the sensitivity solutions are lifted back to high-dimensional space to compute the sensitivities of flow with respect to the design parameters.

This approach has been implemented in a scientific package named OPtimization Toolkit for Highly NOnlinear Systems (OPTHiNOS). Figure 1 shows the optimization procedure schematically. The unsteady sensitivity solution, as

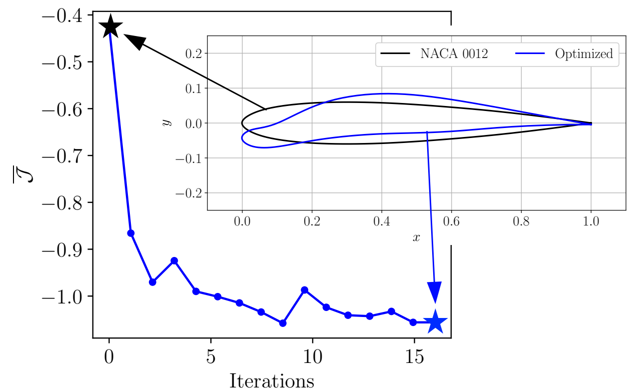

Figure 2: Variations in the time-averaged objective function during the optimization procedure.

the output of the auto-encoder, is used to approximate the sensitivity of the objective function with respect to design variables. Then the design is updated for the next optimization cycle.

4. Results

In the present work, the kinematics of a wing section is considered in cartesian coordinates. The translation and pitching functions of a wing can be defined as

$$\begin{aligned} \mathrm{x}(t) &= \mathrm{z}(t) = 0, \\ \mathrm{y}(t) &= \mathrm{y}_a \sin(2\pi f_a t), \\ \theta(t) &= \theta_0 + \theta_a \sin(2\pi f_a t + \theta_s), \end{aligned} \tag{2}$$

where, y_0 is the plunging amplitude, normally represented in non-dimensional form, y_0/c. Additionally, θ_0 denotes the mean pitching angle, θ_a is the pithing amplitude, and θ_s is the shift angle between the plunging and pitching motions. The pivot point/line is located at the center of pressure $\mathrm{x}_{cp} = 0.25c$ to reduce the effect of aerodynamic loads on the pitching moment.

The present approach is applied to shape optimization of a 2D moving wing section at $Re = 2,400$ and reduced frequency $k = 1.41$, where the non-linear

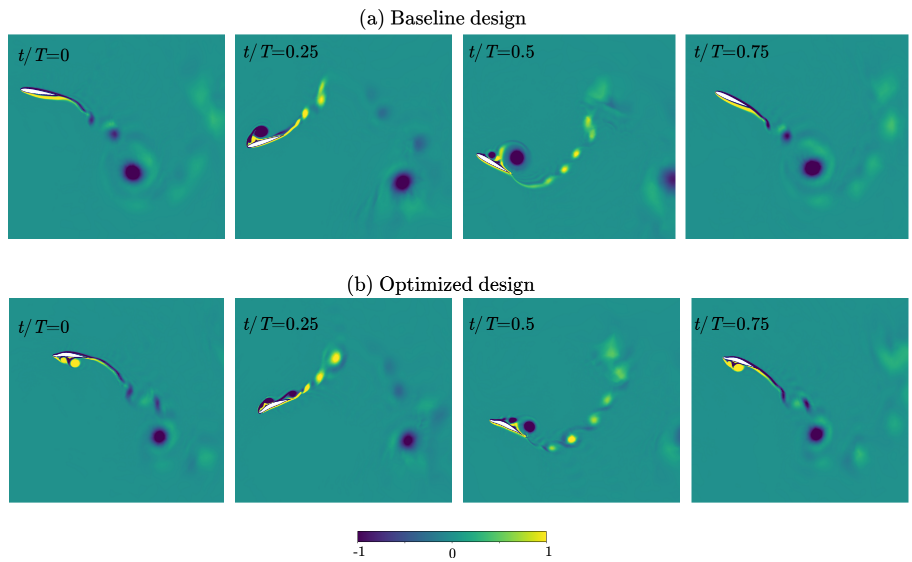

Figure 3: Vorticity contours for the baseline and optimized designs.

interaction of this wing with fluid introduces chaotic flow structures. The phase angle between the pitching and plunging motions is $\theta_s = 90°$, and the plunging amplitude is $\mathrm{y}_a = c$. In general, 11 design parameters are defined to control the shape of the airfoil. Additionally, 30 constraints and 28 bounds are defined to control the shape variations in design space. Figure 2 displays the shape of the baseline (NACA 0012) and the optimized airfoils. The optimized airfoil has more camber with a thinner leading edge, which helps generate more lift. It is shown that the proposed approach can significantly improve the thrust force and propulsive efficiency by 37% and 40%, respectively.

In Figure 3, at instant $t/T^* = 0.25$, Leading Edge Vortex (LEV) for the baseline design is fully developed. However, the optimized wing creates two LEVs simultaneously. One of the LEVs begins to develop from the leading edge, and the other one grows from the wing's upper curvature about $\mathrm{x} = 0.5c$. Additionally, these LEVs remain attached to the upper surface of the wing, which delays stall. Furthermore, at instant $t/T^* = 0.5$, the primary LEV is in the pinch-off mode, and the baseline airfoil is in the post-stall region. However,

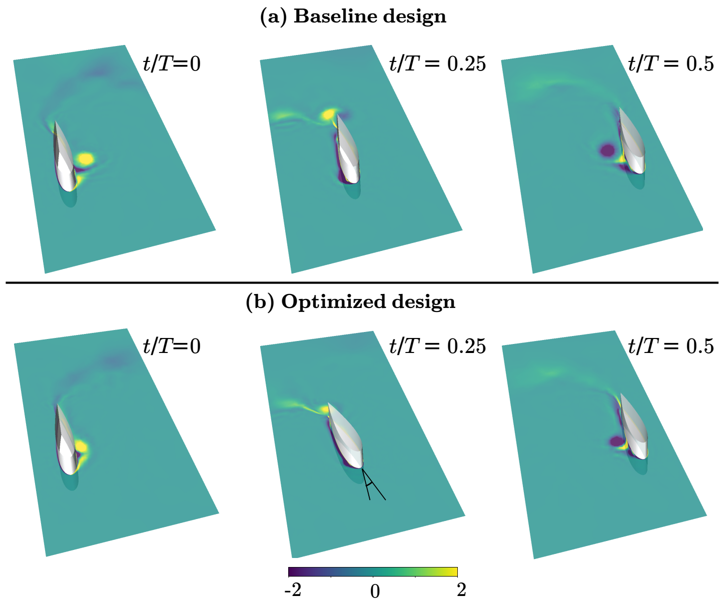

Figure 4: Vorticity contours for the baseline and optimized designs.

the optimized wing still produces higher lift, and breaks down TEVs into smaller vortices.

In the second example, kinematic optimization of a 3D NACA 0020 wing at $Re = 2 \times 10^4$, $\mathrm{y}_a = 0.5$, $k_a = 2$, and $\theta_s = 90°$ is considered. This optimization helps improve the thrust force of a pure plunging motion by combining it with a pitching motion. In this case, the pitching angle is optimized to maximize the thrust force. Solving this problem is impossible by conventional PDE-constrained optimization, due to the chaoticity of the flow in this example. Moreover, applying conventional LSS requires 1.5Tb of memory for solving an extensive system of equations using parallel algorithms. However, sensitivity analysis using the present approach only requires 4Mb of memory, which is achievable on a single core. The optimized pitching angle increases the thurst force by about 29.6%. Figure 4 displays vorticity contours on a plane passing through the middle of the wing. It is shown that the optimized design postpones stall, which leads to higher lift compared to the baseline design.

5. Conclusions

The present approach, which reduces large-scale chaotic problems into a ROM-constrained optimization framework, can be applied to high-fidelity optimization. Therefore, this approach is suitable for aerodynamic optimization using high-fidelity solution techniques, such as LES and DNS. Furthermore, it is expected that using this approach we can design more efficient and cleaner aircraft to meet the emissions regulations.

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