Wing Airfoils Generation Based on a New Parametric Curve for Aerodynamic Optimization Application

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In an ecological context, various measures are sought in order to improve the carbon footprint of the aerospace industry. One way, called the "Morphing-Wing", consists in modify the wing shape continuously with the intention of improving aerodynamic configuration of the aircraft at each flight time. In this context, a new airfoil parameterization method was designed, using 4 Bezier curves globally governed by 17 parameters. All the 17 parameters were linked together and bounded, so that they never led to an invalid airfoil. This airfoil generator allowed the design of a wide range of airfoils including a large proportion of known ones (NACA, Eppler, Wortmann, etc.). Indeed, 88 different airfoils were matching with under 0.5% of error.

Introduction

Nowadays increasingly sensitive to the global warming, the aerospace industry is committed to reduce its toxic gas emissions. Indeed, although aerospace traffic is more efficient than cars traffic, this sector needs to be more ecological in the future, especially because this mode of transport is becoming indispensable in the world and the number of flights should greatly increase in terms of tourist and professional travels. For instance, according to International Civil Aviation Organization (ICAO) emissions calculator [1], more than 18,500 kilogram of fuel is burnt for a single trip connecting San Francisco to New-York. This amount of fuel leads to a considerable emission of 73,000 kilogram of Carbon Dioxide (CO2) in the atmosphere for an aircraft carrying around 250 passengers. To limit these unwanted emissions, the aerospace industry aims to halve aircraft CO2 emissions registered in 2005 before 2050 [2]. As part of this ecological program, some improvements will takes place, such as improvement of commercial airlines trajectories [3-7], engines simulations improvement [8-13], or finally aerodynamic improvement by changing their shapes or by using smooth materials. As part of available aerodynamic optimization, it exist some "Morphing-Wing" technologies aiming to control the shape of an aircraft wing during the flight [14-20]. Indeed, by changing their shapes along the flight, wings could be more efficient for every flight conditions reached by the aircraft [20-23]. In this kind of aerodynamic study, the research could consist in using an optimization process in order to find an optimal airfoil shape.

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An airfoil is composed by two curves, the first one is dedicated to design the inner surface, and the second one the upper surface. By convention, each airfoil surface (the inner and the upper) are characterized using X and Y coordinates. Consequently, it could seem obvious that each coordinates need to be in relation each other and could not be generating independently. For that, in an optimization problem, it is common to use airfoil generator which are based on parameterized curves [24], where, X and Y coordinates will be obtained from equations governed by a set of parameter.

Among technics that can be used as airfoil generator [25]-[26], there is the Joukowski transform Eq. (1) which consists in generating an airfoil in the complex plane (z-plane) by applying the Joukowski transform to a circle in the ξ-plane, where $\xi = \chi + i\eta$ and $z = x + iy$. Using the Joukowski transform as an airfoil generator have some advantages such as the low number of parameters and the fact that any combination gives a good shape for an airfoil. However, this method was not convenient because it was not able to reproduce the shape of many known airfoils (i.e. it cannot generate a large kind of airfoils).

$$z = \xi + \frac{1}{\xi} \tag{1}$$

NACA equations can also be used as an airfoil generator [25]. Those equations lead to generate an airfoil from 4 to 5 parameters (i.e. NACA 4 or 5 digits) that is a good point for an optimization process. This method also allows to generate an airfoil with a good shape regardless of parameters values. However, there is a limit of the diversity of airfoil shape that can be generated using NACA equations (i.e. only NACA airfoil can be generate using these equations).

Finally, two others methods based on parametric polynomial Bezier curves can be used. They are called respectively "*Parsec*" and "*Bezier-Parsec*". Bezier curves allow the user to define a curve that is going to join two extremum points under influence of some control points which coordinate have been precised. *Bezier-Parsec* methods generate an airfoil based on two Bezier curves, a thickness curve and a camber curve that are being added together. More precisely, each curve (i.e. the thickness and the camber) is itself designed using two other curves, one for the leading edge and the second one to design the trailing edge of the airfoil [27]. These two methods (*Bezier-Parsec* and *Parsec*) use respectively from 10 to 15 parameters, that can lead, depending on their arrangement; to an inappropriate airfoil shape (i.e. the shape delivered is very noisy) sometimes due to the curve addition (*Bezier-Parsec*) and sometimes due to the overlapping input parameters.

To conclude, none of the technics works as it is required for an aerodynamic optimization process (because we cannot distinct numerically a bad airfoil from a good airfoil and this aspect can generate some dysfunction in the optimization logic). Consequently, a method was developed in order to design anyways a viable airfoil and implement it in an optimization process.

The methodology used to build our airfoil generator will be presented in the two parts. The following part (section II) will present in detail the methodology used to develop a new airfoil generator. And the final section will present some tests performed in order to validate the method.

II – Methodology: The new parametric curve purposed

The methodology here proposed, will conduct to deliver a good airfoil shape (i.e. without generating noise), for any combination of bounded parameters, while keeping a wide range of airfoil shape.

To design our airfoil generator, a set of 88 witness airfoils have been chosen according to their shape diversities. All the airfoil belonging in the chosen database of airfoil has been superposed in Fig. 1. The database is composed with 49 symmetrical airfoils (the maximum thickness of the database is 15%) and 39 cambered where the biggest camber is 6.7% (AH 79-100C) and 6% (NACA6412).

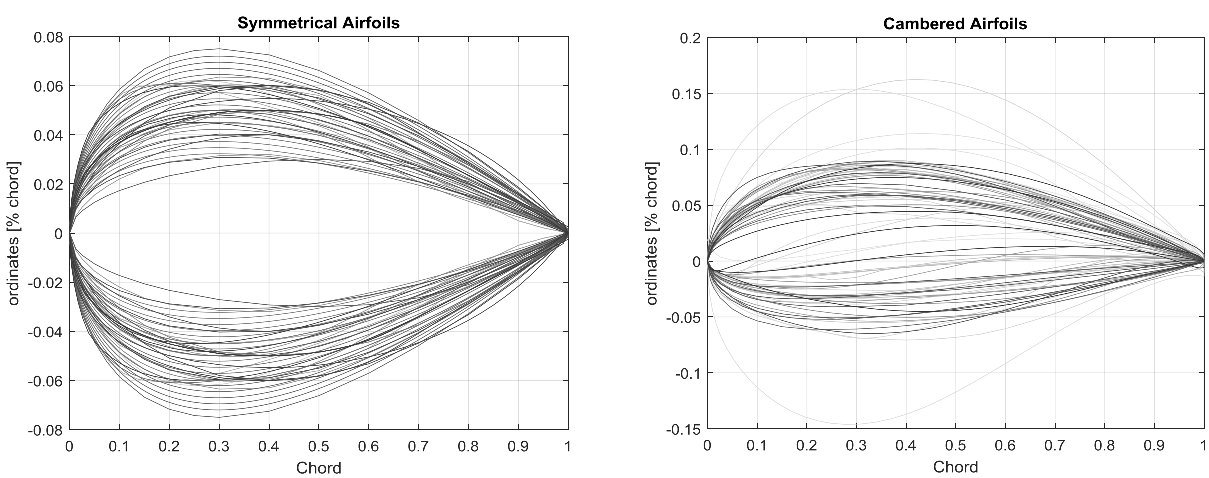

Figure 1. Airfoil database presentation

As previously presented, Bezier curves allow designing a wide range of airfoil but they admit some anomaly on the shape, particularly some noise. As a consequence, we have chosen to base our new parameterized method on a Bezier curve while designing parameters in a way that they cannot generate an incorrect airfoil shape (sharp or noisy).

1) Definition of how many control points are required

The first step of the design methodology consists in finding how many control points is optimal to define, in order to be able to design all airfoil shapes that have been selected in the database of airfoils. More precisely, this step has for goal to analyze the geometrical error observed to match to the 88 airfoils using *n* control points, and select what is the optimal number of control point *n* to generate as much as possible different airfoils. The process shown in Fig. 2 was used to define the optimal *n*. Generally, the process was tested for *n* equal from 4 to 24 control points.

When *n* is set, it is required to define the coordinates of these control point in order to be able to design the airfoil. Depending on the number of control points chosen, it is then required to define their coordinates that are the inputs of the optimization process. X coordinates have been bounded between 0 and 1, and Y coordinates between -0.3 and 0.3. Then, using the Bernstein polynomial we have been able to design an airfoil. The Bernstein polynomial B_n presented in Eq. (2), where *n* is the polynomial order, P_i are the control points, *i* is the iterative index and finally *t* is a vector that defined the Bezier curve length.

$$B_n(t) = \sum_{i=0}^{n} P_i \frac{n!}{i!\ (n-i)!} t^i (1-t)^{n-i} \tag{2}$$

The comparison between the « target » airfoil located in the database and the generated airfoil is measured using the geometric cost $Cost_G$ as shown in Eq. (3).

$$Cost_G = \sum \left(y_{o_{(Upper)}} - y_{BP(Upper)}\right)^2 + \sum \left(y_{o_{(Inner)}} - y_{BP(Inner)}\right)^2 \tag{3}$$

where $y_{o_{(Upper)}}$ and $y_{o_{(Inner)}}$ are ordinates of the upper and the inner surface of the target airfoil, and $y_{BP_{(Upper)}}$ and $y_{BP_{(Inner)}}$ are those corresponding to the airfoil generate by the new generator.

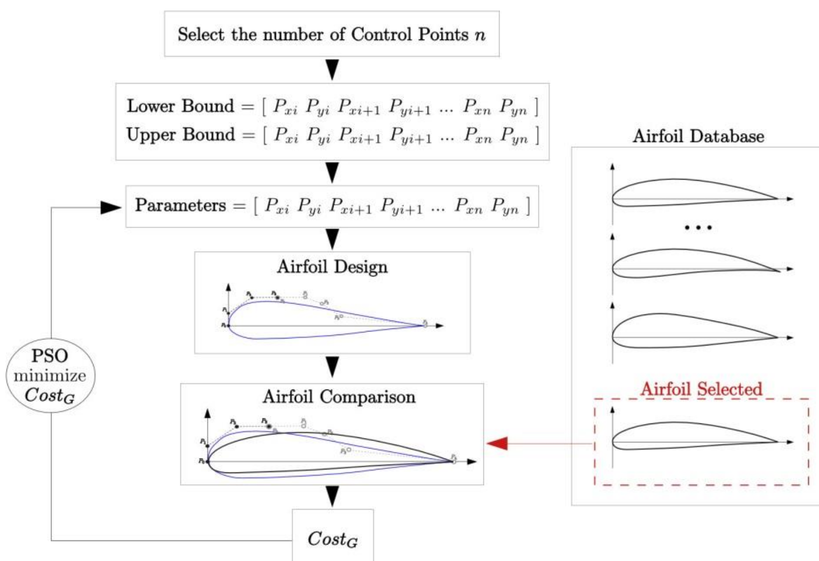

Figure. 2 Process that was used to find how many control points were required to match with the airfoil database

Then a Particle Swarm Optimization algorithm (PSO) is managing the set of coordinate in order to minimize the geometrical difference between the targeted and the generated airfoil (i.e. minimize the $Cost_G$). This process is repeated for all the airfoils of the database (for the same number of control points). At this step, it is possible to compute an average cost $\overline{Cost_G}$ which will be a representative value for each n (i.e. the number of control point) selected.

The Figure 4 shows the average geometric cost $\overline{Cost_G}$ to re-find the 88 airfoils of the database according to the number of the control point allowed in the airfoil generator. It is clear that when the number of control points is increasing, the geometrical error is becoming smaller because there are more points that allows to match airfoil details. In another hand, it is important to limit the number of control points at an average level, in order to keep an advantage both on the large range of generation and on the quick search of a solution.

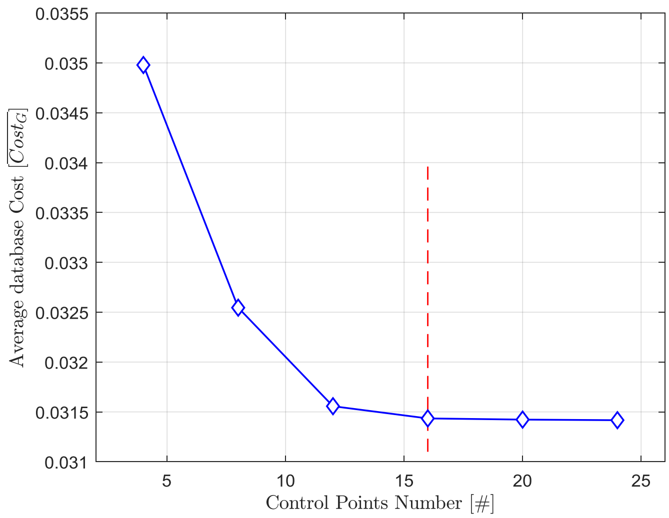

Figure 3. Graphic showing convergence of the average of the geometrical $\overline{Cost_G}$ according to the number of control points

According to the graph displayed in Fig. 3, 16 control points were chosen to re-find the most of airfoil shape while keeping the lowest number of input parameters. 3 control points are dedicated to qualify the leading edge, 3 designs the trailing edge, and the 10 other allows to design the central part of the airfoil. All the control points are allowed to design an unique curve for the upper or the inner surface.

2) Definition of the 16 control points coordinates

In the first step of the design method, we have located each control point directly by their X and Y coordinates. However, to place the 16 control points, it is required to enter the coordinates in X and Y axis of these points, which makes a total of 32 variables to be completed. However 32 inputs imply a too large choice of combination for an optimization algorithm. Indeed, it was decided to express these 32 coordinates with a set of parameters composed by the less number of parameters as possible.

The 17 parameters that were used to express control points coordinates are the 10 angles $\{a_{e1}, a_{e3}, a_{i1}, a_{i3}, a_{e6}, a_{i6}, a_{e7}, a_{i7}, a_{10}, a_{11}\}$ and the 7 length $\{d_5, d_6, x_{e6}, y_{e6}, x_{i6}, y_{i6}, y_t\}$. All of 17 these parameters have been displayed in Fig. 4. Finally, Table 1 explains how all 32 coordinates can be defining with the 17 parameters mentioned previously. Table 1 is organized according to two columns and 2x9 lines corresponding to express control points coordinates of the inner surface and then of the upper surface. An initial point with coordinates (0, 0) has been set as a reference and a common point of both the inner and the upper surface.

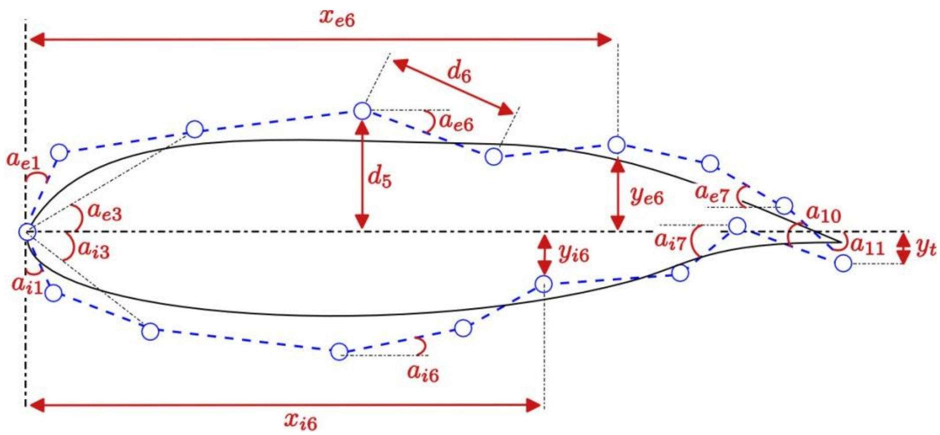

Figure 4. Graphic showing the parameterization of an airfoil using the airfoil generator developed

Table 1. Expression of control points coordinates using inputs parameters of the new airfoil generator

Control points P_i	Abscissas *Px*	Ordinates *Py*
	Inner Surface	
P_{i0}	0	0
P_{i1}	$\sin(a_{i1}) * 0.05$	$-\cos(a_{i1}) * 0.05$
P_{i2}	$\sin(a_{i3}) * 0.1$	$-\cos(a_{i3}) * 0.1$
P_{i3}	0.2	$-\frac{d_5}{2} + d_6$
P_{i4}	$0.2 + 0.2 * \sin\left(\frac{\pi}{2} + a_{i6}\right)$	$-\frac{d_5}{2} + d_6 + 0.2 * \cos\left(\frac{\pi}{2} + a_{i6}\right)$
P_{i5}	$xi6$	$yi6$
P_{i6}	$1 - \sin(a_{10} - a_{i11}) * 0.1 - \sin\left(a_{i7} + \frac{\pi}{2}\right) * 0.1$	$-\cos(a_{10} - a_{i11}) * 0.1 + \cos\left(a_{i7} - \frac{\pi}{2}\right) * 0.1$
P_{i7}	$1 - \sin(a_{10} - a_{i11}) * 0.1$	$-\cos(a_{10} - a_{i11}) * 0.1$
P_{i8}	1	yt
	Upper Surface	
P_{u0}	0	0
P_{u1}	$\sin(a_{e1}) * 0.05$	$\cos(a_{e1}) * 0.05$
P_{u2}	$\sin(a_{e3}) * 0.1$	$\cos(a_{e3}) * 0.1$
P_{u3}	0.2	$\frac{d_5}{2} + d_6$
P_{u4}	$0.2 + 0.2 * \sin\left(\frac{\pi}{2} + a_{e6}\right)$	$\frac{d_5}{2} + d_6 + 0.2 * \cos\left(\frac{\pi}{2} + a_{e6}\right)$
P_{u5}	$xe6$	$ye6$
P_{u6}	$1 - \sin(a_{10} + a_{i11}) * 0.1 - \sin\left(a_{e7} + \frac{\pi}{2}\right) * 0.1$	$\cos(a_{10} + a_{i11}) * 0.1 + \cos\left(a_{e7} - \frac{\pi}{2}\right) * 0.1$
P_{u7}	$1 - \sin(a_{10} + a_{i11}) * 0.1$	$\cos(a_{10} + a_{i11}) * 0.1$
P_{u8}	1	yt

3) Expression of the airfoil shape from the control points location

As previously seen, each surface (i.e. the upper and the inner) are generate using the Bernstein polynomial (see Eq. (2)), under the control of 9 control points. Finally, to design the inner and the upper curve of the airfoil using our new methodology, it is so required to define 4 Bernstein polynomial curves, two that are going to define the upper curve ($X_{B_{8_{upper}}}$, $Y_{B_{8_{upper}}}$) (see Eq. (4)) and the two other for the inner surface ($X_{B_{8_{inner}}}$, $Y_{B_{8_{inner}}}$) (see Eq. (5)) where Px_i are the control points abscissas (first column of Table 1) and Py_i are their ordinates (second column of Table 1). M_8 is the binomial matrix at the 8th order. An 8th order is used because of the 9 control points chosen.

$$\left| \begin{matrix} X_{B_{8_{upper}}}(t) \\ Y_{B_{8_{upper}}}(t) \end{matrix} \right. = \left| \begin{matrix} [Px_0 & Px_1 & \dots & Px_8]_{upper} \\ [Py_0 & Py_1 & \dots & Py_8]_{upper} \end{matrix} \right. \times M_8 \times \begin{bmatrix} t^8 \\ t^7 \\ \vdots \\ 1 \end{bmatrix} \tag{4}$$

$$\left| \begin{matrix} X_{B_{8_{inner}}}(t) \\ Y_{B_{8_{inner}}}(t) \end{matrix} \right. = \left| \begin{matrix} [Px_0 & Px_1 & \dots & Px_8]_{inner} \\ [Py_0 & Py_1 & \dots & Py_8]_{inner} \end{matrix} \right. \times M_8 \times \begin{bmatrix} t^8 \\ t^7 \\ \vdots \\ 1 \end{bmatrix} \tag{5}$$

t is a vector that is moving from 0 to 1 with 200 points, so it generate airfoil with 200 coordinates ($X_{B_{8_{upper}}}$, $Y_{B_{8_{upper}}}$) for the upper surface and 200 coordinates ($X_{B_{8_{inner}}}$, $Y_{B_{8_{inner}}}$) for the inner surface.

III – Results: new parametric curve qualities

In order to examine the efficiency of our method we submitted it to different tests in this results section. For instance, it will be shown parameters bounds used to re-find all the database of airfoils (i.e. the geometric test). Then, an application to an aerodynamic optimization test is presented.

1) Airfoil generator bounds and Geometric research

To be able to generate all the airfoils located in the database, it was required to set upper and lower bounds as described in Table 2.

Table 2. Parameters bounds

Angles bounds										
Parameters	a_{e1}	a_{e3}	a_{i1}	a_{i3}	a_{e6}	a_{i6}	a_{e7}	a_{i7}	a_{10}	a_{11}
LB	-6.0	-9.0	-5.0	-5.0	-20.0	-20.0	-19.9	-32.0	-40.0	0.1
UB	60.0	60.0	60.0	65.0	8.0	8.0	32.0	19.0	40.0	86.0

Length bounds							
Parameters	d_5	d_6	x_{e6}	y_{e6}	x_{i6}	y_{i6}	y_t
LB	-0.05	-0.20	0.30	0.05	0.40	-0.20	-0.02
UB	0.30	0.20	0.80	0.20	0.90	-0.03	0.02

Geometric performances of the airfoil generator that was here developed were tested according to the process shown in Fig. 5. It consists in the selection of a parameters set (bounded using bounds in Table 2), then, generate the corresponding airfoil, and finally, test how much this airfoil match with a targeted airfoil located in the database using the geometric cost $Cost_G$ (see Eq.(3)). A PSO algorithm is in charge of managing the set of parameter in order to minimize $Cost_G$.

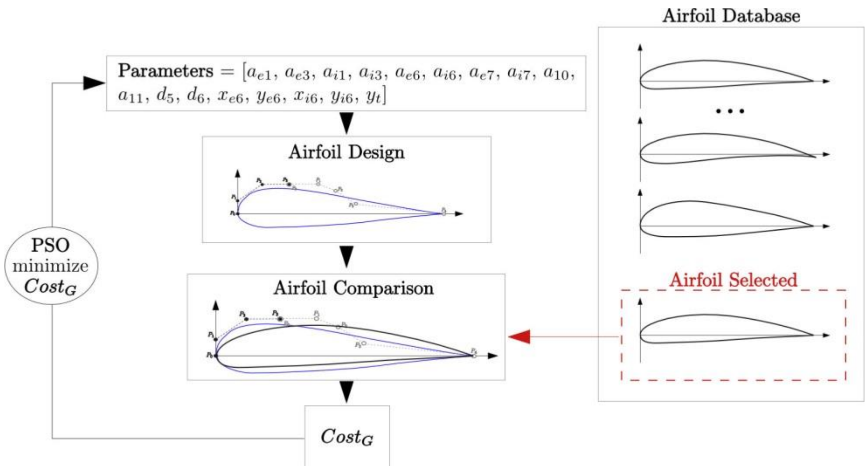

Figure 5. Representation of the geometric research process

The Figure 6 shows the results obtained for this geometric test. On the left of Fig. 6 there is an example of results that can obtained for the airfoil Eppler E168 where the black values indicates the airfoil generated by the new airfoil generator and blue values are corresponding to the original airfoil E168. It is important to mention that the indicated points on the Fig. 6 are corresponding to the airfoil coordinates. On the right on the Fig. 6, it is presented a statistical distribution of the number of airfoils that were found under a given number of errors (relative). For instance, around 77 airfoils have been found by the new airfoil generator with an error below 0.5%.

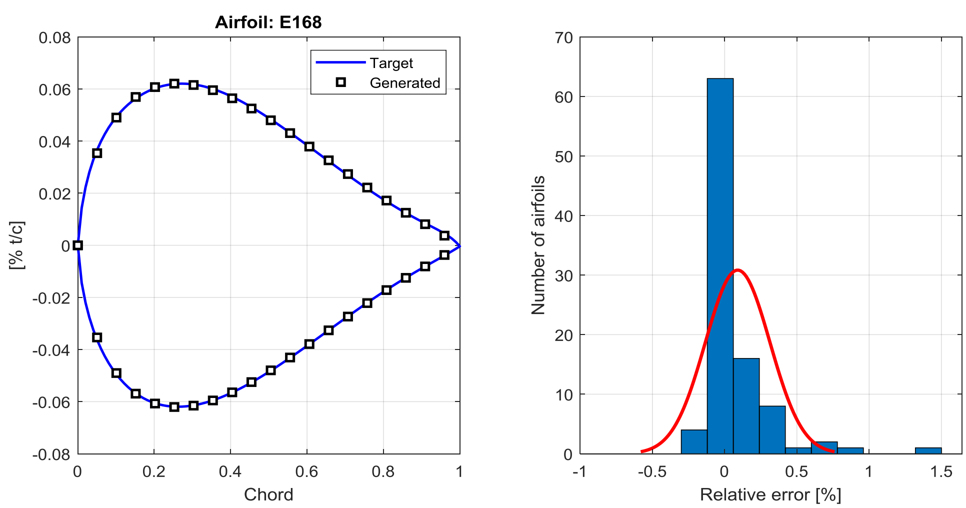

Figure 6. Results of geometric research for the 88 airfoils of the database

The geometric analysis has shown that the new airfoil generator allows to match with all airfoils in the database with a high accuracy. A last test was conducted, it consists in an application of the new airfoil generator in the context of an aerodynamic optimization problem.

2) Application to an aerodynamic optimization problem

Actually, this study as for goal to test if the method can deliver an airfoil shape, that, when it will be adapted on a given wing, the aerodynamic performances of this wing match with targeted aerodynamic coefficient ($C_{L\ target}$, $C_{D\ target}$ and $C_{m\ target}$). In this respect, the optimization process is controlled by an aerodynamic cost $Cost_A$. As described in Eq.(6), $Cost_A$ has no any direct "control" on the airfoil shape.

$$Cost_A = \sum \left(C_{L(target)} - C_L\right)^2 + 10 * \left(C_{D(target)} - C_D\right)^2 + \left(C_{M(target)} - C_M\right)^2 \quad (6)$$

Where, (C_L, C_D and C_m) are aerodynamic coefficients computed using Datcom for an airfoil generated using the developed method (Eq. (4) and (5)) applied on a generic wing shape.

Consequently, this test will measure if the airfoil generator developed in this paper is able to deliver "correct" airfoils shapes in order to allow them to be accepted in an aerodynamic solver. Here Datcom is the solver that was used because it can give to the user very quick results and accurate coefficients because this software, developed by the United State Air Force (USAF) is based on a huge flight test database [28]. The aerodynamic process of optimization is illustrated in Fig. 7.

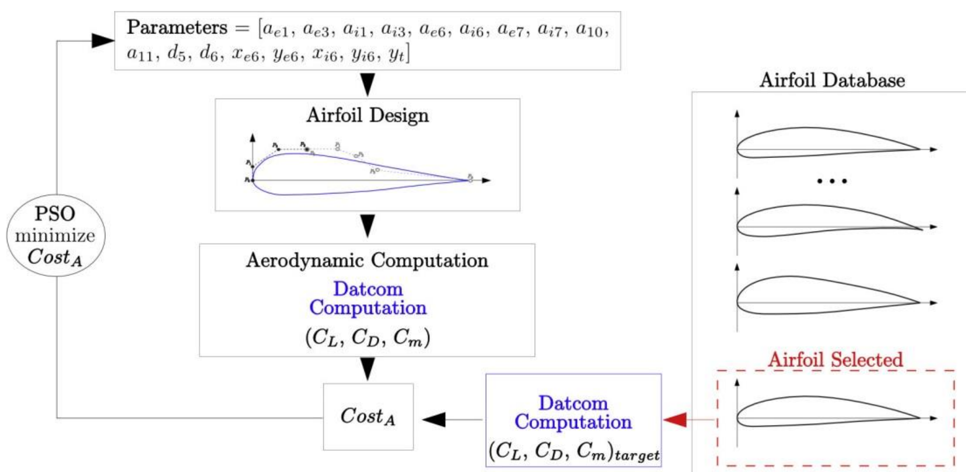

Figure 7. Representation of the aerodynamic research process

The Figure 8 shows some results obtained during this aerodynamic research. On the left of the Fig. 8, there is an example of the results obtained for a symmetrical airfoil (NACA0015), and on the right results obtained for a cambered one (GOE459). It can be seen that aerodynamic coefficients between the targeted airfoil chosen in the database, representing using black squares markers, and generated airfoil, representing using blue curves and "o" markers are extremely close. On the other hand, if we look at the original shape, and the shape found by the algorithm we can see a little difference. Indeed, this difference can come from the aerodynamic solver used (i.e. Datcom). Particularly because this software delivered aerodynamic coefficients from database interpolation, and consequently, some details can be neglected, which can lead that different airfoil shapes can delivered almost same aerodynamic coefficients.

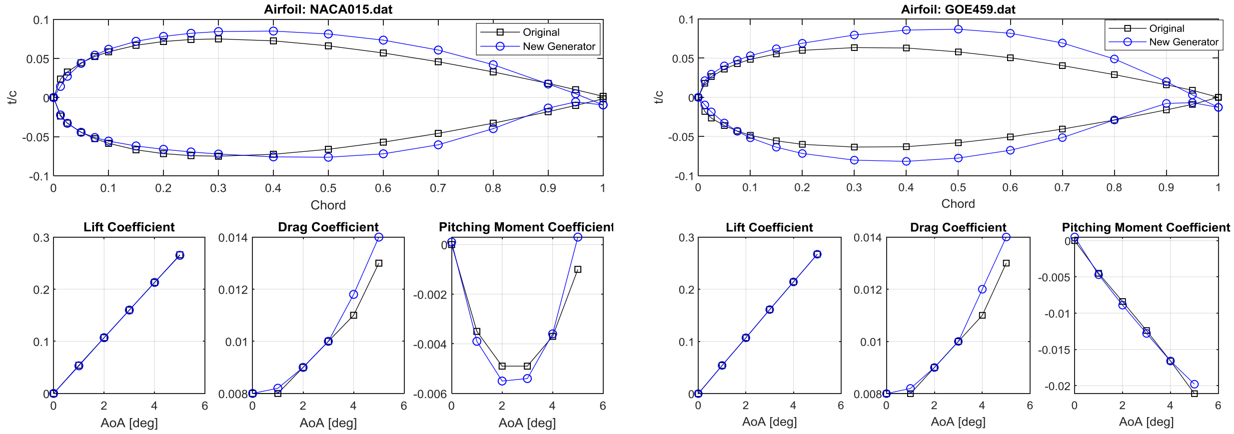

Figure 8. Results of the aerodynamic research

Conclusion

This paper has shown a new method to parameterize an airfoil using Bezier curves. This method has for goal to design a "correct" airfoil shape, without any noise on the geometry in order to use it for an aerodynamic optimization research. Some tests have been performed in order to qualify the airfoil generator performances. The geometric test has shown that the airfoil generator is able to match 77 different airfoils with a maximum difference of 0.5%. Finally, an application in an optimization research in aerodynamic has been perform. The goal consisted in finding the airfoil which, when it is applied on a given wing, it is able to deliver the targeted couple of aerodynamic coefficients (C_L, C_D, and C_m). This study has shown good results using Datcom solver. A small error between the targeted airfoil shape and the generated one is visible. It came from the solver properties, indeed, Datcom makes research in lookup tables to delivered coefficients, consequently, two different shapes (but close shapes) can lead to the same couple of (C_L, C_D, and C_m). It is supposed that if the test can be performed using a high fidelity software, this problem should not appears.

References

[1] ICAO. (2018, April). *Carbon Emissions Calculator*.
Available: https://www.icao.int/environmental-protection/CarbonOffset/Pages/default.aspx

[2] ICAO, "Aviation's Contribution to Climate Change," *ICAO Environmental Report*, 2010.

[3] R. Félix Patrón, A. Kessaci, and R. M. Botez, "Horizontal Flight Trajectories Optimization for Commercial Aircraft through a Flight Management System," *Aeronautical Journal*, vol. 118, 2014.

[4] R. Félix Patrón, Y. Berrou, and R. M. Botez, "Climb, Cruise and Descent 3D Trajectory Optimization Algorithm for a Flight Management System," in *AIAA Aviation Technology, Integration, and Operations, Atlanta, United States*, 2014.

[5] A. Murrieta Mendoza, C. Romain, and R. Botez, "3D Reference Trajectory Optimization for a Commercial Aircraft using a Graph Search Algorithm," in *the MOSIM'16 Innovation in Technology for Performant Systems, Montreal, Quebec, Canada*, 2016.

[6] A. Murrieta-Mendoza, J. Gagné, and R. M. Botez, "New Search Space Reduction Algorithm for Vertical Reference Trajectory Optimization," *INCAS Bulletin*, vol. 8, p. 77, 2016.

[7] A. Murrieta Mendoza, "Application of Metaheuristic and Deterministic Algorithms for Aircraft Reference Trajectory Optimization," École de technologie supérieure, Montreal, Canada, 2017.

[8] M. Zaag and R. M. Botez, "Cessna Citation X Engine Model Identification and Validation in the Cruise Regime from Flight Tests based on Neural Networks combined with Extended Great Deluge Algorithm," in *AIAA Modeling and Simulation Technologies Conference*, 2017, p. 1941.

[9] P.-A. Bardela, "Identification et validation du modèle mathématique du moteur de l'avion d'affaire Cessna Citation X par essais en vol", École de Technologie Supérieure, Montreal, Canada, 2017.

[10] P.-A. Bardela and R. M. Botez, "Identification and Validation of the Cessna Citation X Business Aircraft Engine Component Level Modeling with Flight Tests," in *the AIAA Modeling and Simulation Technologies Conference*, 2017.

[11] P.-A. Bardela, R. M. Botez, and P. Pageaud, "Cessna Citation X Engine Model Experimental Validation.", in *IASTED Modelling, Identification and Control, Innsbruck, Austria*, 2017.

[12] G. Ghazi, R. Botez, and J. M. Achigui, "Cessna Citation X Engine Model Identification from Flight Tests," *SAE International Journal of Aerospace*, vol. 8, pp. 203-213, 2015.

[13] G. Ghazi, A. Mennequin, and R. M. Botez, "Method to Calculate Aircraft Climb and Cruise Trajectory using an Aero-Propulsive Model," in *AIAA Atmospheric Flight Mechanics Conference*, 2017, p. 3550.

[14] O. Sugar Gabor, A. Simon, A. Koreanschi, and R. M. Botez, "Application of a Morphing Wing Technology on Hydra Technologies Unmanned Aerial System UAS-S4," in *The ASME 2014 International Mechanical Engineering Congress & Exposition, Montreal, Quebec, Canada*, 2014.

[15] M. Segui, O. Sugar-Gabor, A. Koreanschi, and R. M. Botez, "Morphing wing application on Hydra Technologies UAS-S4," *Modelling, Identification and Control (MIC),Innsbruck, Austria*, 2017.

[16] T. Grigorie, R. Botez, and A. Popov, "Design and Experimental Validation of a Control System for a Morphing Wing," in *AIAA Atmospheric Flight Mechanics Conference*, 2012.

[17] O. Sugar Gabor, "Validation of Morphing Wing Methodologies on an Unmanned Aerial System and a Wind Tunnel Technology Demonstrator," École de Technologie Supérieure, Montreal, Canada, 2015.

[18] A. V. Popov, T. L. Grigorie, R. M. Botez, Y. Mébarki, and M. Mamou, "Modeling and Testing of a Morphing Wing in Open-Loop Architecture," *Journal of Aircraft*, vol. 47, pp. 917-923, 2010.

[19] O. Ş. Gabor, A. Koreaschi, R. M. Botez, M. Mamou, and Y. Mebarki, "Analysis of the Aerodynamic Performance of a Morphing Wing-Tip Demonstrator Using a Novel Nonlinear Vortex Lattice Method," in *the AIAA Conference, Washington, United States*, 2016.

[20] A. Koreanschi, O. Sugar-Gabor, and R. Botez, "Drag Optimisation of a Wing Equipped with a Morphing Upper Surface," *The Aeronautical Journal*, vol. 120, pp. 473-493, 2016.

[21] O. S. Gabor, A. Koreanschi, and R. M. Botez, "Numerical Study of UAS-S4 Ehécatl Aerodynamic Performance Improvement Using a Morphing Wing Technology," in *Unmanned Systems Canada Annual Conference, Montreal, Quebec, Canada*, 2014.

[22] M. Segui, R. M. Botez, "Cessna Citation X Performances Improvement by an Adaptive Winglet during the Cruise Flight," presented at the *World Academy of Science, Engineering and Technology* conference, *Boston, United States,* 2018.

[23] M. Segui, R. M. Botez, "Cessna Citation X Climb and Cruise Performance Improvement Using Adaptive Winglet," presented at the *Advanced Aircraft Efficiency in a Global Air Transport System*, Toulouse, France, 2018.

[24] H. Sobieczky, "Parametric Airfoils and Wings," in *Recent development of aerodynamic design methodologies*, 1999, pp. 71-87.

[25] I. H. Abbott, A. E. Von Doenhoff, and L. Stivers Jr, "Summary of Airfoil Data," 1945.

[26] F. W. Riegels, "Aerofoil Sections," ed: Butterworths, London, 1961.

[27] R. Derksen and T. Rogalsky, "Bezier-PARSEC: An Optimized Aerofoil Parameterization for Design," *Advances in Engineering Software,* vol. 41, pp. 923-930, 2010.

[28] J.E. Wiliams and S. R. Vukelich "The USAF Stability and Control Digital Datcom. Volume I. User Manual," 1979.